Proof technique:

Without loss of generality, we consider the pure Dirichlet problem ($\gamma_1 = \gamma$). The the error scheme has the form

$$(25)\quad z = u - v : \overline{\omega} \to \mathbb{R}^1 : \quad L_h z(x) = \underset{\text{approximation error}}{\psi(x)} \quad \forall x \in \omega = \overset{\circ}{\omega}$$
$$z(x) = 0 \quad \forall x \in \gamma = \gamma_1$$

Due to the general theory, we have to show

$$\underset{a)\ (26)}{\boxed{\overset{\circ}{W}{}_2^1(\omega) - W_2^{-1}(\omega) - \text{Stability}}} + \underset{b)\ (28)}{\boxed{\text{Approx. } \|\psi\|_{W_2^{-1}(\omega)}}} \Rightarrow \underset{(24)}{\boxed{\text{discrete conv. in } \overset{\circ}{W}{}_2^1(\omega)}}$$

a) $\overset{\circ}{W}{}_2^1(\omega) - W_2^{-1}(\omega)$ - Stability:

- Define discrete $L_2(\omega)$ scalar product

$$(v, z) = (v, z)_{L_2(\omega)} := \sum_{x \in \omega} v(x)\, z(x)\, H(x)$$

- Multiplying (25) with z and estimating from below and above, we get the estimates

$$\tilde{\mu}_1 \|z\|^2_{\overset{\circ}{W}{}_2^1(\omega)} \underset{\boxed{(27)}\ \text{T41 !}}{\leq} (L_h z, z) = (\psi, z) \leq \underbrace{\left[\sup_{w} \frac{|(\psi, w)|}{\|w\|_{\overset{\circ}{W}{}_2^1(\omega)}} \right]}_{=: \|\psi\|_{W_2^{-1}(\omega)}} \|z\|_{\overset{\circ}{W}{}_2^1(\omega)}$$

- Resultat:

$$(26)\quad \boxed{\|z\|_{\overset{\circ}{W}{}_2^1(\omega)} \leq c_s \|\psi\|_{W_2^{-1}(\omega)} \quad \text{with } c_s = \tilde{\mu}_1^{-1}}$$

$$\|L_h^{-1}\|_{L(W_2^{-1}, \overset{\circ}{W}{}_2^1)} \leq c_s = \frac{1}{\tilde{\mu}_1}$$