

SUMMARY of Subseol. 3.4.2: Discrete Convergence of the FVM (T-33 - T44)

$$u: \bar{\omega} \rightarrow \mathbb{R}$$

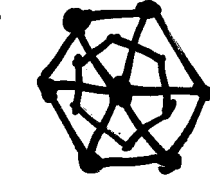
$$A \rightsquigarrow Lu(x) = -\frac{1}{H(x)} \int_{\partial \omega(x)} \alpha(y) \frac{\partial u}{\partial n}(y) dy + \frac{1}{H(x)} \int_{\omega(x)} c(y) u(y) dy$$

$$v = u_h: \bar{\omega} \rightarrow \mathbb{R}$$

$$A_h \rightsquigarrow L_h v(x) = -\frac{1}{H(x)} \sum_{\xi \in S'(x)} \bar{\alpha}(x_\xi) \frac{V(\xi) - V(x)}{h(x_\xi)} S(x_\xi) + \tau(x) v(x) \quad \forall x \in \omega = \bar{\omega}, \quad v(x) = 0 \quad \forall x \in \partial \omega$$

$$D(L) := H^1(\omega) \cap H^1(\bar{\omega}) \subset X = H^1(\Omega) \xrightarrow{L} Y = H^{-1}(\Omega): Lu = b := \frac{1}{H(x)} \int_{\omega(x)} f(y) dy: u \leftarrow E_h u_h$$

$$\begin{array}{c} \text{Restriction} \\ \text{to the} \\ \text{grid } \bar{\omega} \end{array} \quad R_h \downarrow \uparrow \begin{array}{c} \text{e.g. Linear} \\ \text{FE-interp.} \\ Q_h \end{array} \downarrow \uparrow F_h$$



$$X_h = H^1(\omega) \xrightarrow{L_h} Y_h = H^{-1}(\omega): L_h u_h = [f(x)]_{x \in \omega}: u_h \rightarrow R_h u$$

$$\text{Error: } z_h = R_h u - u_h \in X_h = H^1(\omega) = W_2^1(\omega_h): L_h z_h = \tau_h \Leftrightarrow z_h = L_h^{-1} \tau_h$$

$$\Rightarrow \|z_h\|_{X_h} \leq \underbrace{\|L_h^{-1}\|_{L(X_h, X_h)}}_{\leq C_S} \cdot \underbrace{\|\tau_h\|_{Y_h}}_{\leq C_A(\omega)h} \leq C_S C_A(\omega)h$$

Stability +

Approximation $\Rightarrow$ discrete convergence

$$\text{with } \tau_h(u) := L_h R_h u - Q_h A u = -\frac{1}{H(x)} \left[\sum_{\xi \in S'(x)} \bar{\alpha}(x_\xi) \frac{u(x_\xi) - u(x)}{h(x_\xi)} S(x_\xi) - \int_{\omega(x)} c(y) u(y) dy \right] + \dots$$