5. For the special case "Poisson Equation" ($a \equiv 1$, $c \equiv 0$), we have

a) $$-\sum_{j \in \bar{\omega}_h} \int_{\partial \mathcal{H}(x^{(j)})} \frac{\partial u}{\partial n} \bar{v}\, ds = \int_{\Omega} \nabla^T u \nabla v\, dx$$

$\forall u \in \mathring{V}_{0h} = \mathring{P}_1(\mathcal{T}_\Delta)\ \forall v \in \mathring{V}_{0h},\ v \leftrightarrow \bar{v} \in \mathring{T}_{0h} = \mathring{P}_0(\mathcal{T}_{\mathcal{H}})$

$\Longrightarrow\ \bar{a}(u, \bar{v}) = a(u, v)\ \forall u, v \in \mathring{V}_{0h},\ v \leftrightarrow \bar{v} \in \mathring{T}_{0h}$

b) $\Longrightarrow$ $\underset{\text{FVM}}{K_B} = \underset{\text{FEM}}{K_L}$, but, in general, $\underline{f}_B \neq \underline{f}_L$! ⟸ (mms)

c) <u>Error estimates</u>: $u \in V_0 \cap S_0^1$ ($|||\cdot||| \simeq \|\cdot\|_1 \simeq |\cdot|_1$)

(12) $|||u - u_B||| \le \tilde{c} \inf\limits_{v_h \in \mathring{V}_{0h}} |||u - v_h|||$

Estimates (11) and (12) immediately yield

(13) $|||u - u_L||| \le |||u - u_B||| \le \tilde{c}\, |||u - u_L|||$

If $u \in H^2(\Omega)$ or, at least, $u \in H^2(\delta_r)$ $\forall r \in R_h$, $\forall h \in \Theta$, then (12) and the Approximation Theorem give

(14) $$\boxed{|u - u_B|_{1,\Omega} \le \tilde{c}\, a_{12}\, h\, |u|_{2,\Omega}}$$

or

(14) $$\boxed{|u - u_B|_{1,\Omega} \le \tilde{c}\, a_{12} \Big(\sum_{r \in R_h} h_r^2 |u|_{2,\delta_r}^2\Big)^{1/2}}$$

6. We can conclude from (13) resp. (11) that the a-posteriori estimates for the FEM, i.e.

$$|||u - u_L||| \le c\, \eta(u_L)$$

are also valid for the FVM, i.e.

$$|||u - u_B||| \le c\, \eta(u_L) \approx c\, \eta(u_B).$$