Remark 3.6:

1. $(7)_{Box\,a)}$ Find $\underline{u}_B \in \mathbb{R}^{N_h}$: $\sum_{i \in \omega_h} u^{(i)} \bar{a}(p^{(i)}, x^{(k)}) = (f, x^{(k)})_0 \;\forall k \in \omega_h$

   Derive the explicit form $(\underline{7})_{B\,a)}$!

2. $K_B = K_B^T$ p.d. (mms) SPD

3. In general, the Line- and area integrals arising in the computation of the entries of K_B must be calculated numerically (see also Subsect. 3.3.1!):

   $\int_{\partial \mathcal{X}(x^{(k)})} \ldots ds \approx$ by quadrature formulas

   $\int_{\mathcal{X}(x^{(k)})} \ldots dx \approx$ by quadrature formulas

4. Using 2nd STRANG's Lemma, we can derive error estimates in the energy norm

   $|||\cdot|||^2 := a(\cdot,\cdot) \approx |\cdot|_1^2 \approx \|\cdot\|_1^2$ on $\overset{\circ}{V}_0 = \overset{\circ}{H}{}^1(\Omega)$

   under the assumption that

   $$u \in V_0 \cap S_0^1(T_\Delta) := \{ v \in \overset{\circ}{V}_0 : (\sum_{r \in R_h} h_r^2 \|\Delta u\|_{0,r}^2)^{1/2} \le c \inf_{v_h \in \overset{\circ}{V}_{0h}} |u - v_h|_1 \}$$

   with positive constants $c = \text{const} \ne c(h)$:

$$(9) \quad |||u - u_B||| \le c \inf_{T_{0h} \ni \bar{v} \leftrightarrow v \in \overset{\circ}{V}_{0h}} \{ |||u - v||| + \|u - \bar{v}\|_{0,\Omega} \} \le c\,h \|u\|_2 .$$

approximation theorem
Ass.: $u \in H^2(\Omega)$ (Ch. 2)

Since (FE-solution)

$$(10) \quad |||u - u_L||| \le \inf_{v \in \overset{\circ}{V}_{0h}} |||u - v|||,$$

we obviously

$$(11) \quad |||u - u_L||| \le |||u - u_B||| \le c \{ |||u - u_L||| + \|u - \bar{u}_L\|_0 \}.$$