Derivation of the System of FV-Equations: $K_B \underline{u}_B = \underline{f}_B$

$(7)_{Box}$ B)

> Find $u_B \in \bar{V}_{0h}$: $\bar{a}(u_B, \bar{v}) = (f, \bar{v})_0 \quad \forall \bar{v} \in T_{0h}$
>
> B) Lumped Galerkin-Petrov Scheme

$$u_B(x) = \sum_{i \in \omega_h}' u^{(i)} p^{(i)}(x)$$

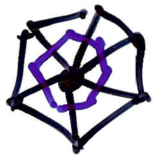
$$\updownarrow \qquad \longleftrightarrow \quad \underline{u}_B = [u^{(i)}]_{i \in \omega_h} \in \mathbb{R}^{N_h}$$

$$\bar{u}_B(x) = \sum_{i \in \omega_h}' u^{(i)} \chi^{(i)}(x)$$

$$\bar{v} = \chi^{(k)}, \quad k \in \omega_h$$

$$-\sum_{j \in \omega_h}' \int_{\partial \mathcal{R}(x^{(j)})} a(x) \frac{\partial u_B}{\partial n}(x) \chi^{(k)}(x)\, ds + \int_\Omega c(x) \bar{u}_B(x) \chi^{(k)}(x)\, dx = \int_\Omega f \chi^{(k)}\, dx$$

$$-\int_{\partial \mathcal{R}(x^{(k)})} a(x) \frac{\partial u_B}{\partial n}(x)\, ds + \int_{\mathcal{R}(x^{(k)})} c(x) \bar{u}_B(x)\, dx = \int_{\mathcal{R}(x^{(k)})} f(x)\, dx$$



$$\sum_{\substack{i \in \omega_h \\ i \in \omega_h(k) \\ x^{(i)} \in S(x^{(k)})}}' u^{(i)} \Big[-\int_{\partial \mathcal{R}(x^{(k)})} a(x) \frac{\partial p^{(i)}}{\partial n}(x)\, dx + \underbrace{\int_{\mathcal{R}(x^{(k)})} c(x) \chi^{(i)}(x)\, dx}_{= \int_{\mathcal{R}(x^{(k)})} c(x)\, dx \cdot \delta_{ki}} \Big] = \int_{\mathcal{R}(x^{(k)})} f\, dx$$

Lumping!

Result:

$(7)_{Box}$ B)

> Find $\underline{u}_B = [u^{(i)}]_{i \in \omega_h} \in \mathbb{R}^{N_h}$: $K_B \underline{u}_B = \underline{f}_B$
>
> $$\sum_{i \in \omega_h(k)}' \underbrace{\Big[-\int_{\partial \mathcal{R}(x^{(k)})} a(x) \frac{\partial p^{(i)}}{\partial n}(x)\, ds + \int_{\mathcal{R}(x^{(k)})} c(x)\, dx \cdot \delta_{ki} \Big]}_{=: K_{ki}} u^{(i)} = \underbrace{\int_{\mathcal{R}(x^{(k)})} f(x)\, dx}_{=: f^{(k)}}$$
>
>