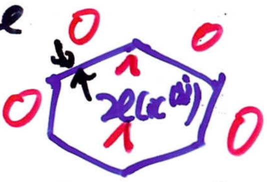
Galerkin-Petrov Approach to the FVM:

Define the test spaces

$$T_h := P_0(\mathcal{T}_\chi) = \Big\{ \bar v = \bar v_h = \sum_{i\in\bar\omega_h}{}' \, v^{(i)} \chi^{(i)}(x) \Big\} \subset L_2(\Omega) \not\subset V = H^1(\Omega)$$

characteristic function of the box $\mathcal{H}(x^{(i)})$

$$\chi^{(i)}(x) = \chi_{\mathcal{H}(x^{(i)})}(x) := \begin{cases} 1, & x \in \mathcal{H}(x^{(i)}) \cup \partial\mathcal{H}^-(x^{(i)}) \\ 0, & \text{otherwise} \end{cases}$$



$$T_{0h} := \mathring{P}_0(\mathcal{T}_\chi) = \Big\{ \bar v(x) = \sum_{i\in\omega_h}{}' \, v^{(i)} \chi^{(i)}(x) \Big\} \subset L_2(\Omega) \not\subset V_0 = \mathring{H}^1(\Omega)$$

$$\bar v(x) = 0 \quad \forall x \in \Gamma$$

Idea: → starting point = strong formulation $(7)_{SF}$

① $\displaystyle\int_\Omega \underbrace{PDE}_{(7)_{SF}} \cdot \bar v \, dx \quad \forall \bar v \in T_{0h}$

$$\sum_{\substack{j\in\bar\omega_h \\ (j\in\omega_h)}}{}' \int_{\mathcal{H}(x^{(j)})} (-\mathrm{div}(a\nabla u) + cu)\, \bar v \, dx = \int_\Omega f \bar v \, dx \quad \forall \bar v \in T_{0h}$$

② Partial Integration in the main part:

$$-\sum_{j\in\omega_h}{}' \int_{\partial\mathcal{H}(x^{(j)})} a(x) \frac{\partial u}{\partial n}(x)\, \bar v(x) \, ds + \sum_{j\in\omega_h}{}' \int_{\mathcal{H}(x^{(j)})} c u \bar v \, dx = \underbrace{\int_\Omega f \bar v \, dx}_{=: (f, \bar v)_0}$$

③ Find $u = u_h \in \bar V_{0h} = \underbrace{\mathring{P}_1(\mathcal{T}_\Delta)}_{\text{FE-space}}$: ②