$$|\ell(u) - \ell(u_h)| = |\langle F, w-w_h\rangle - a(u_h, w-w_h)|$$

$$\overset{(1)}{\le} \sum_{r\in R_h} \|R_r(u_h)\|_{0,\delta_r} \|w-w_h\|_{0,\delta_r} + \sum_{e\in E_h} \|R_e(u_h)\|_{0,e} \|w-w_h\|_{0,e} \tag{48}$$

$$= \sum_{r\in R_h} \underbrace{\Big[\|R_r(u_h)\|_{0,\delta_r} \|w-w_h\|_{0,\delta_r} + \frac{1}{2}\sum_{e\in\partial\delta_r\setminus\Gamma_D} \|R_e(u_h)\|_{0,e}\|w-w_h\|_{0,e}\Big]}_{=:\,\eta_{\delta_r}(u_h)}$$

$$= \sum_{r\in R_h} \eta_{\delta_r}(u_h) = \eta(u_h),$$ no constants, but w, w_h! dual weights

PDE: $Lu = f$ in Ω — $-\Delta u = f$ in Ω Example

BC: $\ell u = g$ on Γ — $u = 0$ on Γ

$R_r(u_h) = f - Lu_h = f + \Delta u_h$ PDE residual

$R_e(u_h) = [\frac{\partial u}{\partial N}]_e = [\frac{\partial u_h}{\partial n}]_e$ jumps in the fluxes

<u>Question:</u> How to determine the dual weights $w \in \tilde{V}_0$ and its FE approximation $w_h \in \tilde{V}_{0h}$?

① We proceed as in the proof of Theorem 2.19, i.e.

1) $w_h = \text{CLÉMENT}(w)$: $\|w-w_h\|_{0,\delta_r} \le c h_r |w|_{1,\omega(\delta_r)}$ ...

2) Find FE-solution $\tilde{w}_h \in \tilde{V}_{0h}$: (47) $\Rightarrow |\tilde{w}_h|_{1,\omega(\delta_r)} \approx |w|_{1,\omega(\delta_r)}$

② We solve the "dual" problem (47) twice:

$w_h \in \tilde{V}_{0h}$: $(47)_h$

$\tilde{w}_h \in V_{0h}^{HO}$: $(47)_h \approx w$

$\Big\}$ $\|w-w_h\|_\bullet \approx \|\tilde{w}_h - w_h\|_\bullet$

③ Solve $(47)_h \Rightarrow w_h \in \tilde{V}_{0h}$

$w \approx$ Higher-order reconstruction $(w_h) = \tilde{w}_h$ $\Big\}$ — " —