The typical application of the 1st Lemma of STRANG

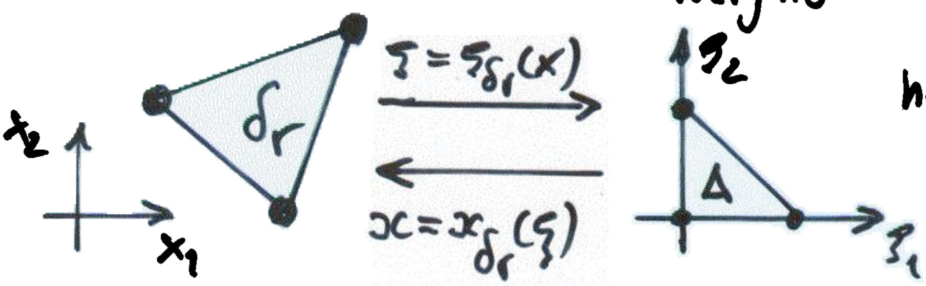
= Analysis of the effects of numerical integration:

Let us consider again the model problem from Subsection 2.2.1 with the data $a=0, \varkappa=0, g_1=g_2=g_3=0$:

- $V = H^1(\Omega)$, $V_0 = \{v \in V : v = 0 \text{ on } \Gamma_1\} = \tilde{V}_g$,
- $a(u,v) = \int_\Omega \lambda(x) \nabla^T u(x) \nabla v(x)\, dx$

$$\Big\downarrow$$

$$a_h(u_h, v_h) = \sum_{r \in R_h} \sum_{\beta \in S_r} \underset{\substack{\uparrow \\ \text{weights}}}{w_r^{(\beta)}} \left(\square \right)_{\xi = \underset{\substack{\uparrow \\ \text{integration points}}}{\xi_r^{(\beta)}}} \cdot \underbrace{\tfrac{1}{2}}_{\text{meas } \Delta}$$



here: $S_r = \{1\}$
$\xi_r^{(1)} = \left(\frac{1}{3}, \frac{1}{3}\right)$
$w_r^{(1)} = 1$

where $(\square) = \lambda(x_{\delta_r}(\xi))\, J_{\delta_r}^{-T} \nabla_\xi^T u_h(\cdot)\, J_{\delta_r}^{-T} \nabla_\xi v_h(\cdot)\, |J_{\delta_r}|$

- $\langle F, v \rangle := \int_\Omega f(x)\, v(x)\, dx$

$$\Big\downarrow$$

$$\langle F_h, v_h \rangle_h := \sum_{r \in R_h} \sum_{\beta \in S_r} w_r^{(\beta)} f(x_{\delta_r}(\xi_r^{(\beta)}))\, v_h(x_{\delta_r}(\xi_r^{(\beta)}))\, |J_{\delta_r}| \frac{1}{2}$$

■ $\boxed{\text{E 2.13}}$ Formulate conditions imposed on the data
10 $\lambda \in L_\infty(\Omega) \cap W_?^?(\delta_r)$, $f \in L_2(\Omega) \cap W_?^?(\delta_r)$ $\forall r \in R_h$ $\forall h \in \Theta$
↓ and on the quadrature rule (i.e. algebraic
20 exactness for $\int_\Delta \varphi(\xi)\, d\xi$) such that
↓ $\|u - \tilde{u}_h\|_{1,\Omega} \le c(u, f, \lambda)\, h$, where $\tilde{u}_h : (\tilde{1})_h$
30 Can you already ensure the $O(h)$ accuracy
for Δ ($S_r = \{1\}$, $w_r^{(1)} = 1$ $\xi_r^{(1)} = (1/3, 1/3)$)!