

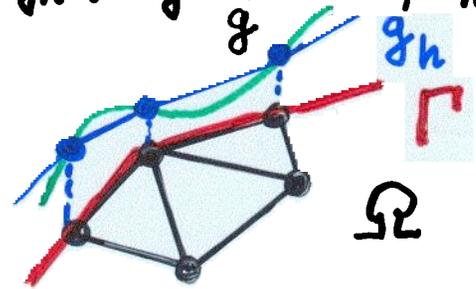
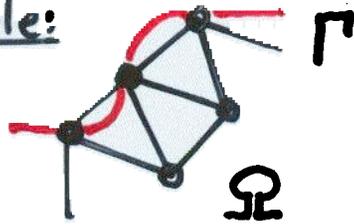
■ The practice often forces us to violate the standard approach (= variational principle):

⇒ Variational Crimes:

1. Numerical Integration:  $\int \rightarrow \sum$   
 $\mathcal{I}_h(\cdot)_h: a(\cdot, \cdot) \rightarrow a_h(\cdot, \cdot)$  5  
 $\langle F, \cdot \rangle \rightarrow \langle F_h, \cdot \rangle_h$  4  $\downarrow (\tilde{1})_h$

2. 1st Kind BC cannot always be fulfilled precisely in  $V_h$ ,  
 i.e.  $V_{0h} \not\subset V_0$  X and/or  $V_{gh} \not\subset V_g$  X even if  $V_h \subset V$ !

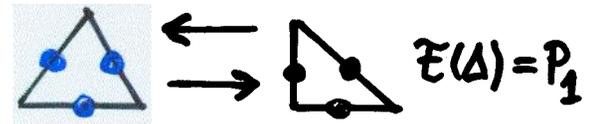
Example:



3. Conformity of the elements is violated:

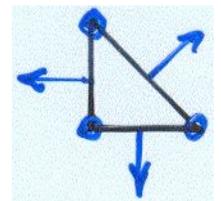
(in particular, for 4th order PDEs:  $V = H^2(\Omega)$ !),  
 i.e.  $V_h \not\subset V$  1 (e.g. the use of  $C^0$ -el. for 4th order PDEs)

Examples: a) 2nd order PDEs: Crouzeix-Raviart-el.



b) 4th order PDEs: Morley-element

$F(\Delta) = P_2$



4. Assumptions (33)<sub>2a)+2b)</sub> 6 have to be completed by additional conditions imposed on the discrete bilinear form  $a_h(\cdot, \cdot)$   
 ⇒ (34) resp. (39) !