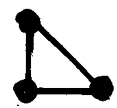
Remark 2.14:

1. If the function u fulfils the stronger regularity assumption
$$u \in V_g \cap W_\infty^{k+1}(\Omega),$$
then the interpolation ($\Rightarrow$ approximation) error estimates
$$|u - \mathrm{int}_{V_h} u|_{W_\infty^s(\Omega)} \leq c h^{k+1-s} |u|_{W_\infty^{k+1}(\Omega)}$$
are valid, where $s = 0, 1$ ($W_\infty^0(\Omega) = L_\infty(\Omega)$).

<u>Proof</u>: In analogy to (32) with $\ell \in [W_p^{k+1}(\Omega)]^*$ and then $p \to \infty$!

2. <u>Question:</u> $\| u - u_h \|_{W_\infty^s(\Omega)} \leq$? $(k = 0, 1)$ provided that $u \in V_g \cap W_\infty^{k+1}(\Omega)$?

<u>Answer:</u> non-trivial!

(a) For $d = 2$, $k = 1$, $\mathcal{F}(\Delta) = P_1$: $\Delta =$ ,
i.e. Linear triangular element = Courant's elem.:
$$\| u - u_h \|_{L_\infty(\Omega)} \leq c h^2 |\log h|^{3/2} |u|_{W_\infty^2(\Omega)},$$
$$\| u - u_h \|_{W_\infty^1(\Omega)} \leq c h |\log h| |u|_{W_\infty^2(\Omega)}$$

<u>Techniques for proving such estimates:</u>

① Method of weighted Sobolev spaces proposed by NITSCHE (1975)

② Method of discrete Green's functions proposed by SCOTT (1975)

<u>Literature:</u> CIARLET

(b) For "all" (?) other cases, one can prove quasioptimal L_∞- resp. W_∞^1- discretization error estimates ($\hat{=}$ approximation error estimates)!