5. A-priori estimate of $|u|_{k+1,\Omega}$ resp. $\|u\|_{k+1,\Omega}$ by the input data ($=W_2^{k+1}$-coercitivity):
e.g. Dirichlet problem for the Poisson equation in a convex domain $\Omega \subset \mathbb{R}^2$ *:

<u>CF:</u> $$-\Delta u = f \text{ in } \Omega$$
$$u = 0 \text{ on } \Gamma = \partial\Omega$$

<u>VF:</u> Find $u \in V_g = V_0 = \overset{\circ}{W}{}_2^1(\Omega)$:
$$\int_\Omega \nabla^T u \, \nabla v \, dx = \int_\Omega f \cdot v \, dx \quad \forall v \in V_0,$$
with given $f \in L_2(\Omega)$

<u>H^2-coercitivity:</u>
It follows from $f \in L_2(\Omega)$, $\Omega \subset \mathbb{R}^2$ - bounded, Lip, convex:

$$\boxed{\begin{array}{l} 1.\ \exists!\ u \in V_0 \cap H^2(\Omega): \\ 2.\ |u|_{2,\Omega} \le \underset{\uparrow\, c=1}{1} \cdot \|f\|_{0,\Omega} \end{array}}$$

<u>Proof</u> mms*

$$\int_\Omega f^2 dx = \int_\Omega (-\Delta u)(-\Delta u)\, dx \overset{\text{integr. by parts}}{=} \ldots$$