Remark 2.9:

1. If, instead of (9) $\|J_{\delta_r}\| \le c_2 h_r$, the more general conditions (22) $|\partial^\beta x_{\delta_r,i}(\xi)| \le \bar{c}_2 h_r^{|\beta|}$ $\forall |\beta| \le k+1$ are assumed, then the error estimate (23') holds:

$$(23')\quad \|u-u_h\|_{1,\Omega} \underset{b)}{\le} c_{1,k+1}\Big[\sum_{r\in R_h} h_r^{2k}\|u\|^2_{k+1,\delta_r}\Big]^{1/2} \underset{a)}{\le} c_{1,k+1} h^k \|u\|_{1,\Omega}$$

2. If $u \in V_g \cap W_2^{\ell}(\Omega)$ with $1 \le \ell \le k+1$ ($\ell \in \mathbb{R}$), then

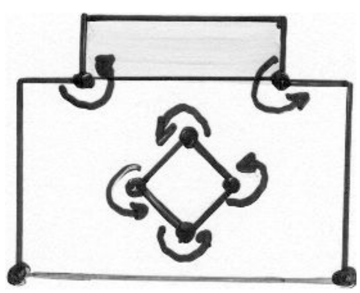
$$\|u-u_h\|_{1,\Omega} \le \frac{\mu_2}{\mu_1}\bar{a}_{1,k+1} h^{\ell-1}\|u\|_{\ell,\Omega}.$$

3. If $u \in W_2^{\ell_r}(\delta_r)$, $1<\ell_r \le k+1$, $\forall r \in R_h$ $\forall h \in \circledast$, then

$$\|u-u_h\|_{1,\Omega} \le \frac{\mu_2}{\mu_1}\Big[\sum_{r\in R_h} \bar{a}_{1,\ell_r} \underbrace{h_r^{2(\ell_r-1)}}_{} \|u\|^2_{\ell_r,\delta_r}\Big]^{1/2}$$

mesh grading: $\approx h^{2k}$

4. Our Example: $d=2$, $k=1$, $\mathcal{F}(\Delta)=P_1$: $\Delta =$ 
$x_{\delta_r}(\cdot) \in P_1$



$$u \in W_2^{1+s_r}(\delta_r),\ 0 < s \le s_r \overset{\bullet}{\le} 1$$
$$s_r = s_r(\partial\Omega, BC, \dots) = s_r(\cdot)$$
$$s = s(\partial\Omega, BC, \dots) = s(\cdot)$$

$$\Rightarrow \|u-u_h\|_{1,\Omega} \le \frac{\mu_2}{\mu_1}\Big[\sum_{r\in R_h} \bar{a}_{1,1+s_r} h_r^{2s_r}|u|^2_{1+s_r,\delta_r}\Big]^{1/2}$$

$$\le \frac{\mu_2}{\mu_1}\bar{a}_{1,1+s} h^{s}\Big[\sum_{r\in R_h}|u|^2_{1+s_r,\delta_r}\Big]^{1/2}$$