Theorem 2.25: (Embedding for $W_p^1(\Omega)$)

Ass.: $\Omega \subset \mathbb{R}^d$ * $\wedge$ Lip, $1 < p, q < \infty$,

St.: Statements 1. - 3. of Th. 2.24 remain valid if $\mathring{W}_p^1(\Omega)$ is replaced by $W_p^1(\Omega)$.

Theorem 2.26: (Embedding for $W_p^k(\Omega)$)

Ass.: $\Omega \subset \mathbb{R}^d$ * $\wedge$ Lip, $1 < p, q < \infty$,
$0 \leq j < k$, $k = 1, 2, 3, \ldots$

St.: 1. $W_p^k(\Omega) \hookrightarrow C^j(\bar{\Omega})$ if $(k-j)p > d$.

2. Let $(k-j)p \leq d$.
Then the embeddings
$W_p^k(\Omega) \subset W_q^j(\Omega)$ resp. $\mathring{W}_p^k(\Omega) \subset \mathring{W}_q^j(\Omega)$
are
- continuous if $q \leq q_* := \frac{pd}{d-(k-j)p} \wedge q < \infty$,
- compact if $q < q_*$.

The results on the embedding of $\mathring{W}_p^k(\Omega) \subset \mathring{W}_q^j(\Omega)$ don't need the assumption that Ω is Lip.

Remarks:

1. St. 3 of Th. 2.24 and Th. 2.25 can easily be generalited to $W_p^k(\Omega)$, i.e.
$W_p^k(\Omega) \subset W_q^j(\Omega_m)$ if (mms)
$W_p^k(\Omega) \hookrightarrow W_q^j(\Omega_m)$ if (mms)
2. All results are sharp!
For instance: $d=2$: $H^2(\Omega) \hookrightarrow C(\bar{\Omega})$, BUT
$H^1(\Omega) \not\subset C(\bar{\Omega})$!

Proofs: Lecture Notes "Numerik I", Sect. 3.8, pp. 80-86.