Theorem 2.22:

Ass.: $\Omega \subset \mathbb{R}^d$: $\dagger \wedge$ Lip, $\Gamma = \partial\Omega$

$\tilde{\Omega} \subset \mathbb{R}^d$: $\ast \wedge \overline{\Omega} \subset \tilde{\Omega}$

$1 \le p \le \infty$, $k = 0, 1, \dots$

St.: Then there exists (not unique!) a Linear, bounded extension operator

$$\Pi \in L(W_p^k(\Omega), \mathring{W}_p^k(\tilde{\Omega})),$$

i.e. $v = \Pi u$: 1. $v|_\Omega = u$

2. $\|v\|_{W_p^k(\tilde\Omega)} = \|\Pi u\|_{W_p^k(\tilde\Omega)} \le c_E \|u\|_{W_p^k(\Omega)}$.

Proof: see Lit., e.g. [Michlin] and Exercise 2.24 ∎

Exercise 2.23: Let

$$\Omega = \{(x,y) \in \mathbb{R}^2 : a < x < b,\ 0 < y < c\}$$

$$\overline{\tilde{\Omega}} = \overline{\Omega} \cup \overline{\Omega}_- \quad \text{, with}$$

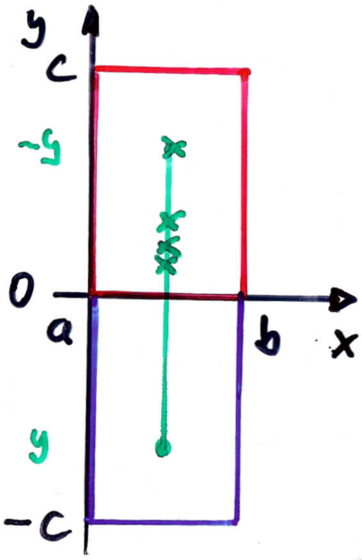
$$\Omega_- = \{(x,y) \in \mathbb{R}^2 : a < x < b,\ 0 > y > -c\}$$

$$\Omega \subset \tilde{\Omega}$$

$\exists!$ (mms) $k+1$ reals $\lambda_1, \dots, \lambda_{k+1}$:

$$(-1)^\ell \lambda_1 + \left(-\tfrac{1}{2}\right)^\ell \lambda_2 + \dots + \left(-\tfrac{1}{k+1}\right)^\ell \lambda_{k+1} = 1$$

$\ell = 0, 1, \dots, k$



Show that the Hestenes extension

$$v(x,y) = \begin{cases} u(x,y) & , 0 \le y \le c \\ \lambda_1 u(x,-y) + \dots + \lambda_{k+1} u(x, -\frac{y}{k+1}) & , -c \le y \le 0 \end{cases}$$

is really an extension of u under keeping the classes $\mathcal{B} = C^k, W_2^\ell, \ell = 0, 1, \dots, k+1$.