Further Direct Consequences from (14):

1. Formula of Integration by Parts (IbyP):

$$(15)\quad \int_\Omega \partial_i u \cdot v \, dx = -\int_\Omega u \partial_i v \, dx + \int_\Gamma u v n_i \, ds$$

$\forall u \in W_p^1(\Omega)$, $\forall v \in W_q^1(\Omega)$, $\frac{1}{p}+\frac{1}{q}=1$, $1<p<\infty$

Proof: Set $w = u \cdot v \in W_1^1(\Omega)$ in (14). ■

2. The 1st Green's Formula for $(-\Delta)$:

$$(16')\quad \int_\Omega \nabla^T u \cdot \nabla v \, dx = -\int_\Omega \Delta u \cdot v \, dx + \int_\Gamma \frac{\partial u}{\partial n} \cdot v \, ds$$

$\forall u \in W_2^2(\Omega)$ $\forall v \in W_2^1(\Omega)$

Proof: Set $w = \partial_i u \cdot v \in W_1^1(\Omega)$ in (14) $\wedge \sum_{i=1}^{d}$ ■

3. The 2nd Green's Formula for $(-\Delta)$:

$$(16'')\quad \int_\Omega (\Delta u \cdot v - u \Delta v) \, dx = \int_\Gamma \frac{\partial u}{\partial n} v \, ds - \int_\Gamma u \frac{\partial v}{\partial n} \, ds \quad \forall u,v \in H^2(\Omega).$$

Proof: follows immediately from (16') changing u and v ■

4. The 1st Green's Formula for Δ^2:

$$(16''')\quad \int_\Omega \Delta u \Delta v \, dx = \int_\Omega \Delta^2 u \cdot v \, dx - \int_\Gamma \partial_n \Delta u \cdot v \, ds + \int_\Gamma \Delta u \, \partial_n v \, ds$$

$\forall u \in W_2^4(\Omega)$ $\forall v \in W_2^2(\Omega)$, $\partial_n = \partial/\partial n$

Proof: $\int_\Omega \partial_i^2 u \, \partial_i^2 v \, dx$ two times IbyP ■

5. The IbyP-formula (15) yields (mms)

$$\int_\Omega \operatorname{curl}(u) \cdot v \, dx = \int_\Omega u \cdot \operatorname{curl}(v) \, dx - \int_\Gamma (u \times n) \cdot v \, ds.$$

$\forall u, v \in H(\operatorname{curl}) \cap [C^1(\bar\Omega)]^d \quad (d=3)$