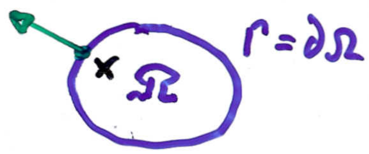
2.5. The Formula of Integration by Parts and other Integration Formulas

- **The main formula of the Differential and Integral Calculus (DIC):**

  - $d=1$

$$\int_a^b w'(x)\,dx = w(x)\Big|_a^b = w(b)-w(a) \quad \forall w \in C^1[a,b]$$



  - $d \in \mathbb{N}$

$$(14) \qquad \int_\Omega \partial_i w\,dx = \int_\Gamma w\cdot n_i\,ds \quad \forall w \in C^1(\overline{\Omega}),$$

where $n = n(x) = (n_1(x),\dots,n_d(x))^T$ – exterior unit normal,
$|n| = 1$, $n_i(x) = \cos\sphericalangle(n(x), \vec{x}_i)$.

  - Inserting $w = u\cdot v$ with $u, v \in C^1(\overline{\Omega})$ into (14) and using the product rule give the classical formula of integration by parts:

$$(15) \qquad \int_\Omega \partial_i u\cdot v\,dx = -\int_\Omega u\,\partial_i v\,dx + \int_\Gamma u v n_i\,ds \quad \forall u,v \in C^1(\overline{\Omega})$$

  - **Lemma 2.18:**

    Ass.: $\Omega \subset \mathbb{R}^d$ ∧ Lip, $d \in \mathbb{N}_0$ (formal)

    St.: Then the main formula of the DIC is also valid for functions $w \in W_1^1(\Omega)$:

$$(14) \qquad \boxed{\int_\Omega \partial_i w\,dx = \int_\Gamma w\cdot n_i\,ds \quad \forall w \in W_1^1(\Omega)}$$