3) $v \in \mathcal{R}_0$, i.e. $v = \text{const} \wedge 0 = f_1(v) = \left(\int_{\Gamma_1} |v|^p ds\right)^{1/p} = |v|\,|\Gamma_1|^{1/p}$

$\Rightarrow v \equiv 0$.

Th. 2.13 implies that $\exists\, \underline{c}, \overline{c} = \text{const} > 0$:

$$(10)\quad \underline{c}\left(\int_{\Gamma_1} |u|^p ds + \int_\Omega |\nabla u|^p dx\right)^{1/p} \le \left(\int_\Omega (|u|^p + |\nabla u|^p)\,dx\right)^{1/p} \le \overline{c}\left(\int_{\Gamma_1}|u|^p ds + \int_\Omega |\nabla u|^p dx\right)^{1/p}$$

$$\forall u \in W_p^1(\Omega)$$

For functions $u \in \mathring{V}_0 \subset W_p^1(\Omega)$, we get from (11)

$$(11)\quad \int_\Omega |u|^p dx \le \overline{c}^{\,p} \int_\Omega |\nabla u|^p dx \quad \forall u \in \mathring{V}_0.$$

q.e.d.

- **<u>Corollary 2.16:</u>**
The $W_p^1(\Omega)$ - semi-norm $|\cdot|_{W_p^1(\Omega)}$ is a norm on $\mathring{V}_0 = \{ v \in W_p^1(\Omega) : \gamma_{\Gamma_1} u = 0\}$ which is equivalent to the standard $W_p^1(\Omega)$-norm $\|\cdot\|_{W_p^1(\Omega)}$:

$$(10)\quad \underline{c}\,|u|_{W_p^1(\Omega)} \le \|u\|_{W_p^1(\Omega)} \le \overline{c}\,|u|_{W_p^1(\Omega)} \quad \forall u \in \mathring{V}_0 \subset W_p^1(\Omega)$$

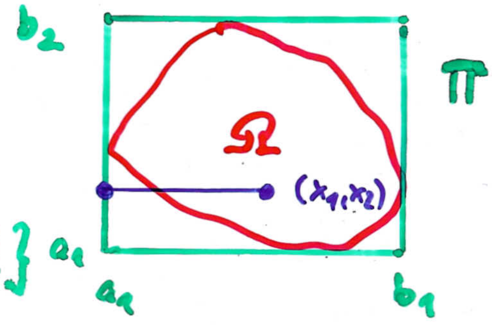
- **<u>Exercise 2.17:</u>** (Constr. Proof of Friedrichs' ineq.)
Show that $\exists\, c_F = \text{const} > 0$: $(\Gamma_1 = \Gamma,\ p = 2)$

$$(11)\quad \int_\Omega (u(x))^2 dx \le c_F^2 \int_\Omega |\nabla u(x)|^2 dx \quad \forall u \in \mathring{V}_0 = \overset{\circ}{H}{}^1(\Omega)$$

with $c_F = \frac{1}{\sqrt{2}} \min_{i=\overline{1,d}} (b_i - a_i)$,

where

$$\Omega \subset \Pi := \{ x = (x_1, \ldots, x_d) \in \mathbb{R}^d : a_i < x_i < b_i,\ i = \overline{1,d} \}$$



<u>Hint:</u>

$$u(x_1, x_2) = u(a_1, x_2) + \int_{a_1}^{x_2} \frac{\partial u}{\partial \xi_1}(\xi_1, x_2)\, d\xi_1.$$