(9)

Two Special Function Spaces: → H-spaces

- $H(\mathrm{div}) = H(\mathrm{div},\Omega) := \{u \in [L_2(\Omega)]^d : \exists\, \mathrm{div}\, u \in L_2(\Omega)\}$
  = H-space with the scalar product
  $(u,v)_{H(\mathrm{div})} := (u,v)_{L_2(\Omega)} + (\mathrm{div}\, u, \mathrm{div}\, v)_{L_2(\Omega)}$,
  $\|u\|_{H(\mathrm{div})} := (u,u)^{1/2}_{H(\mathrm{div})}$.
  What about trace and inverse trace theorems ?
- $H(\mathrm{curl}) = H(\mathrm{curl},\Omega) := \{u \in [L_2(\Omega)]^d : \exists\, \mathrm{curl}\, u \in [L_2(\Omega)]^d\}$
  = H-space with the scalar product
  $(u,v)_{H(\mathrm{curl})} := (u,v)_{L_2(\Omega)} + (\mathrm{curl}\, u, \mathrm{curl}(v))_{L_2(\Omega)}$,
  $\|u\|_{H(\mathrm{curl})} := (u,u)^{1/2}_{H(\mathrm{curl})}$.
  What about trace and inverse trace theorems !

Sobolev-Slobodeckij-Spaces:

- **Def. 2.11:** $H^s(\Omega) = W_2^s(\Omega)$, $s \in \mathbb{R}$ (H-spaces)
  1) $s = k$ = integer $\in \mathbb{Z}$: $H^s(\Omega) := W_2^s(\Omega)$ defined by Def. 2.7 ($s=k \geq 0$) and Def. 2.9 ($s = k < 0$).
  2) $s > 0$: $s = k+\sigma$, $k \in \mathbb{N}_0$, $\sigma \in (0,1)$:
     $H^s(\Omega) := \{u \in H^k(\Omega) : |u|_{H^s(\Omega)} < \infty\}$ with
     $\|u\|^2_{H^s(\Omega)} = (u,u)_{H^s(\Omega)}$ (separable H-space)
     $(u,v)_{H^s(\Omega)} = (u,v)_s := (u,v)_{H^k(\Omega)} + (u,v)_{k+\sigma}$,
     $|u|^2_{H^s(\Omega)} := (u,u)_{k+\sigma}$
$$(u,v)_{k+\sigma} := \sum_{|\alpha|=k} \int_\Omega \int_\Omega \frac{(\partial^\alpha u(x) - \partial^\alpha u(y))(\partial^\alpha v(x) - \partial^\alpha v(y))}{|x-y|^{d+2\sigma}}\, dx\, dy$$
  3) $s < 0$: $H^s(\Omega) := [\overset{\circ}{H}{}^{-s}(\Omega)]^*$, $\overset{\circ}{H}{}^{-s}(\Omega) = \overline{C_0^\infty(\Omega)}^{\|\cdot\|_{-s}}$
- Sobolev-Slobodeckij-spaces on manifolds, e.g. $H^s(\Gamma)$, $\Gamma = \partial\Omega$
- $H^{1/2}(\Gamma) \simeq \gamma_0 H^1(\Omega)$