⑦

- Traces and trace spaces:
  - Let $\Omega \subset \mathbb{R}^d$ be ✱ and Lip, $1 \le p < \infty$. The there exists a linear, continuous operator

$$(9)_0 \begin{cases} \gamma_0 \in L(W_p^1(\Omega), L_p(\Gamma)): & \Gamma = \partial\Omega \\ \gamma_0 u(x) = u(x) \ \forall x \in \Gamma = \partial\Omega \ \forall u \in C^1(\bar{\Omega}), \\ \|\gamma_0 u\|_{L_p(\Gamma)} \le c \|u\|_{W_p^1(\Omega)} \ \forall u \in \overline{C^1(\Omega)}^{\|\cdot\|_{W_p^1}} = W_p^1(\Omega) \end{cases}$$

The function $g = \gamma_0 u$ is called generalized trace function of u on Γ.
γ_0 is called trace operator. The trace operator γ_0 as operator from $W_p^1(\Omega)$ to $L_p(\Gamma)$ is not surjective, i.e. not every function $g \in L_p(\Gamma)$ is a trace of some function $u \in W_p^1(\Omega)$!

  - Trace space for the space $H^1(\Omega)$:

$$H^{1/2}(\Gamma) := \gamma_0 H^1(\Omega) \subset L_2(\Gamma),$$

$$(9)_{1/2} \qquad \|g\|_{H^{1/2}(\Gamma)} := \inf_{\substack{u \in H^1(\Omega): \\ \gamma_0 u = g}} \|u\|_{H^1(\Omega)}$$

  - Trace theorem:

$$(9)_t \qquad \exists c_t = \text{const} > 0: \ \|\gamma_0 u\|_{H^{1/2}(\Gamma)} \le c_t \|u\|_{H^1(\Omega)} \ \forall u \in H^1(\Omega)$$

  - Inverse trace theorem: (extension theorm)

$$(9)_e \qquad \exists c_e = \text{const} > 0: \ \forall g \in H^{1/2}(\Gamma) \ \exists u \in H^1(\Omega):$$

$$\gamma_0 u = g \text{ and}$$

$$\|u\|_{H^1(\Omega)} \le c_e \|g\|_{H^{1/2}(\Gamma)}$$

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