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2.2. The Sobolev Spaces $W_p^k(\Omega)$ and some Elemantary Properties

■ <u>Repetition:</u> L_p - spaces (= Banach - spaces)

$L_p(\Omega) := \{u : \Omega \to \mathbb{R}^1 \text{ - measurable} : \|u\|_{L_p(\Omega)} < \infty\}$,
where $1 \le p < \infty$ and

$$\|u\|_{L_p(\Omega)} := \left(\int_\Omega |u|^p dx\right)^{1/p}.$$

$L_\infty(\Omega) := \{u : \Omega \to \mathbb{R}^1 \text{ - measurable} : \|u\|_{L_\infty(\Omega)} < \infty\}$,
where $\|u\|_{L_\infty(\Omega)} := \operatorname{ess\,sup}_{x \in \Omega} |u(x)|$.

$L_2(\Omega)$ = Hilbert - space (H - space)

$$\|u\|_{L_2(\Omega)} = (u, u)^{1/2}_{L_2(\Omega)},$$

$$(u, v)_{L_2(\Omega)} = \int_\Omega u(x) v(x)\, dx$$

<u>Dual spaces:</u> $X^* = X'$

$[L_p(\Omega)]^* \equiv [L_p(\Omega)]' = L_q(\Omega)$, $\frac{1}{p} + \frac{1}{q} = 1$, $1 < p < \infty$.

$[L_1(\Omega)]^* = L_\infty(\Omega)$

$[L_\infty(\Omega)]^* \neq L_1(\Omega)$, i.e. $L_1(\Omega)$ is not the dual space of $L_\infty(\Omega)$!