- **Def. 2.5:** Distributive derivatives

Let $u \in D'(\Omega)$. Then the relation

$$(8) \qquad \langle u_\alpha, \varphi \rangle := (-1)^\alpha \langle u, \partial^\alpha \varphi \rangle \quad \forall \varphi \in D(\Omega)$$

obviously defines a distribution $u_\alpha \in D'(\Omega)$ for all $\alpha = (\alpha_1, \dots, \alpha_d)$, $\alpha_i \in \mathbb{N}_0$, $i = \overline{1,d}$ (mms).
We write

$$\partial^\alpha u = u_\alpha \in D'(\Omega),$$

and call $\partial^\alpha u$ $\alpha$th distributive derivative of $u$.
A distribution obviously has distributive derivatives of arbitrary order!

- Distributive derivatives vs Sobolev's derivatives:

If the distributive derivative $\partial^\alpha u \in D'(\Omega)$ of a locally integrable function $u \in L_{loc}(\Omega)$ is regular, i.e. $\partial^\alpha u \in L_{loc}(\Omega)$, then there exists Sobolev's derivative in the sense of Def. 2.1 and both derivatives can be identified:

$$\langle \partial^\alpha u, \varphi \rangle := (-1)^{|\alpha|} \langle u, \partial^\alpha \varphi \rangle \quad \forall \varphi \in D(\Omega)$$

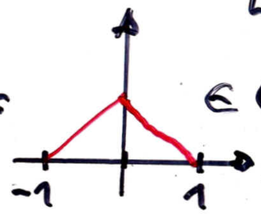
$$\| $$

$$\int_\Omega \partial^\alpha u \, \varphi \, dx := (-1)^{|\alpha|} \int_\Omega u \, \partial^\alpha \varphi \, dx \quad \forall \varphi \in C_0^\infty(\Omega)$$

- **Example 2.6:**

Let us consider the function

$$u(x) = \begin{Bmatrix} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le +1 \end{Bmatrix} =$$




$$\begin{array}{c} D'(\Omega) \\ \cup \\ L_{loc}(-1,1) \\ \cup \\ L_p(-1,1) \\ \cup \\ \in C[-1,1] \end{array}$$

a) $u' = \partial^1 u \in$ ?

b) $u'' = \partial^2 u \in$ ?

c) $u''' = \partial^3 u \in$ ?

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