2. Useful Tools from the Theory of Sobolev's Spaces [1) 2)]

2.1. Sobolev's Generalized Derivatives and Distributions

- Let $\Omega \subset \mathbb{R}^d$ be a bounded (*) Lip domain with the boundary $\Gamma = \partial\Omega$, $\Omega \neq \emptyset$, $d \in \mathbb{N}$ $(d = 1, 2, 3)$.
  Define
  $\overset{\circ}{C}{}^\infty(\Omega) :=$ space of all continuously infinite times differentiable real functions with compact support in Ω,
  and introduce the following <u>convergence</u> in $\overset{\circ}{C}{}^\infty(\Omega)$:

(1) $\boxed{\varphi_n \xrightarrow[n\to\infty]{} 0 \text{ in } \overset{\circ}{C}{}^\infty(\Omega) \text{ iff a) } \exists K \subset \Omega \text{ - compact} : \varphi_n(x) = 0 \ \forall x \notin K \ \forall n \in \mathbb{N}, \text{ b) } \partial^\alpha \varphi_n \underset{n\to\infty}{\rightrightarrows} 0 \text{ on } K \ \forall \alpha}$

where $\partial^\alpha \varphi = \dfrac{\partial^{|\alpha|}\varphi}{\partial x_1^{\alpha_1} \ldots \partial x_d^{\alpha_d}}$, $\alpha = (\alpha_1, \ldots, \alpha_d)$, $\alpha_i \in \mathbb{N}_0$, $|\alpha| = \alpha_1 + \ldots + \alpha_d$.

(2) $\varphi_n \xrightarrow{n\to\infty} \varphi$ iff $\varphi - \varphi_n \xrightarrow{n\to\infty} 0$ in $\overset{\circ}{C}{}^\infty(\Omega)$.

- $D(\Omega) := \overset{\circ}{C}{}^\infty(\Omega)$ equipped with the convergence (1)
  $=$ space of all fundamental functions

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1) To a great extend this Chapter should be a repetition from classes on PDEs or NuPDEs!

2) See also Chapter 3 of "Numerik I" that can be downloaded from the NuMa homepage: