2. Estimates (18) or (18') immediately yield:

$$(18'')\quad \inf_{v_h\in V_h} \|u-v_h\|_{1,\Omega} \le \bar{a}_{1,k+1}\, h^{k} \begin{cases} |u|_{k+1,\Omega} & \text{Th. 2.6.}\\ \|u\|_{k+1,\Omega} & \text{Rem. 2.7.}\end{cases}$$

with $\bar{a}_{1,k+1}^2 = a_{1,k+1}^2 + a_{0,k+1}^2 h^2$.

3. Estimates (18), (18'), (18'') are optimal w.r.t. the h-power (cf. E 2.9), i.e. $\exists\, u \in W_2^{k+1}(\Omega)$: $\inf_{v_h\in V_h} |u-v_h|_{s,\Omega} \ge c\, h^{k+1-s}$

with some h-independent positive constant c.

4. If $u\in W_2^{\ell}(\Omega)$, $1<\ell\le k+1$ ($\ell\in\mathbb{R}$ real!), then we have the estimate

$$\inf_{v_h\in V_h} |u-v_h|_{s,\Omega} \le a_{s,\ell}\, h^{\ell-s}\, \|u\|_{\ell,\Omega}.$$

5. If $u\in W_2^{\ell_r}(\delta_r)$, $1<\ell_r\le k+1$, $\forall r\in R_h$ $\forall h\in\Theta$, then we have the estimate

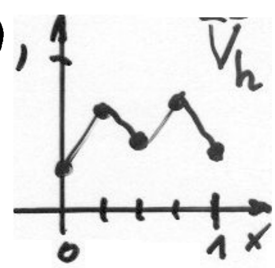
$$\inf_{v_h\in V_h} |u-v_h|_{s,\Omega} \le \Big(\sum_{r\in R_h} a_{s,\ell_r}\, h_r^{2(\ell_r-s)}\, \|u\|^2_{\ell_r,\delta_r}\Big)^{1/2}$$

$s=0,1$ or $s\in[0,1]$

can be used for a-priori mesh grading!

$\boxed{\text{E 2.9}}$ Show that, for $d=1$, $\Omega=(0,1)$, $k=1$: $\mathcal{F}(\Delta)=P_1$, $u(x)=x^2$, we have

$$\inf_{v_h\in V_h} \int_0^1 |u'-v_h'|^2\,dx = \frac{1}{3}h^2$$



$\boxed{\text{E 2.10}}$ Show the completeness of the family of the FE-spaces $\{V_{0h}\}_{h\in\Theta}$ in V_0!