$$\delta_r \quad x = x_{\delta_r}(\xi) \quad \Delta \quad \xi = \xi_{\delta_r}(x) \quad \delta_r$$

- element — map to → $\boxed{\text{master element}}$ — map back to → element:

$$\|\bullet\|^2_{1,\Omega} = \sum_r \|\bullet\|^2_{1,\delta_r} = \sum_r \int_{\delta_r} [\ldots\ldots] dx =$$

$$\overset{x = x_{\delta_r}(\xi)}{\stackrel{\downarrow}{=}} \sum_r \int_\Delta [\ldots ? \ldots] \, |J_{\delta_r}| \, d\xi$$

$$\leq \sum_r c h^{?} \underbrace{\|\bullet_\xi\|^2_{1,\Delta}}_{} \leq \ldots \overset{\xi = \xi_{\delta_r}(x)}{\stackrel{\downarrow}{\leq}} ? \, h^{?} \, ?$$

estimation of the interpolation error
on the master element Δ
with the help of (→ Subsect. 2.4.1)
the BRAMBLE/HILBERT-Lemma

- <u>Result:</u> = a-priori error estimate
with respect to (w.r.t.) the $\|\cdot\|_1$-norm
(→ see Subsections 2.4.2 and 2.4.3)

- A-priori error estimates are also interesting w.r.t. other norms:
  - L_2-norm $\|\cdot\|_{L_2(\Omega)} = \|\ \|_{0,\Omega}$ → 2.4.4
  - L_∞-norm $\|\cdot\|_{L_\infty(\Omega)} = \|\ \|_{0,\infty,\Omega}$ → 2.4.5
  - W^1_∞-norm $\|\cdot\|_{W^1_\infty(\Omega)} = \|\cdot\|_{1,\infty,\Omega}$ → 2.4.5
  - L_p-norm $\|\cdot\|_{L_p(\Omega)} = \|\cdot\|_{0,p,\Omega}$ → Literature
  - W^1_p-norm $\|\cdot\|_{W^1_p(\Omega)} = \|\cdot\|_{1,p,\Omega}$ → Literature

  $\vdots$

- Goal-oriented estimates: $l \in V_0^*$ or $l \in (V_0 \cap H^2)^*$:
$|l(u - u_h)| \leq ? \, h^{?} \, ?$ $\widehat{=}$ a-priori
⇝ a-posteriori error estimates !!! → Sect. 2.6.