

- Essential BC = 1st Kind BC: $u = g_1$ on Γ_1 ($\bar{V}_{g_1}, \bar{V}_{0h}$)

1st Version: Homogenization

- $u_j = g_1(x^{(j)}) \quad \forall j \in \delta_h := \bar{\omega}_h \setminus \omega_h$
- update the RHS: $f^{(i)} = f^{(i)} - \sum_{j \in \delta_h} K_{ij} g_1(x^{(j)})$
 $\forall i \in \omega_h \quad (\forall i \in \bar{\omega}_h)$
- cancel the columns with the indices $j \in \delta_h$
- cancel the rows with the indices $i \in \delta_h$

2nd Version: Penalty technique

$$K_{ii} := 10^s, \quad f^{(i)} := 10^s g_1(x^{(i)}) \quad \forall i \in \delta_h$$

$$\Rightarrow u_i = g_i - 10^{-s} \sum_{j \neq i} K_{ij} u_j \quad \text{with } s\text{-suff. large!}$$

this corresponds to 3rd Kind BC with $\alpha = 10^s$!

3rd Version: $K_{ji} = K_{ij} = \delta_{ij} \quad \forall i \in \delta_h \quad \forall j \in \omega_h$ and $f^{(i)} = g_1(x^{(i)}) \quad \forall i \in \delta_h$

- RESULT: System of FE equations for determining the unknown nodal values $u^{(i)}$, $i \in \omega_h$:

(i) Considering 1st Kind BC via 1st version:

$$\underline{(4)}_h \text{ Find } \underline{u}_h = [u^{(i)}]_{i \in \omega_h} \in \mathbb{R}^{N_h}: K_h \underline{u}_h = \underline{f}_h \text{ in } \mathbb{R}^{N_h}$$

(ii) Considering 1st Kind BC via 2nd or 3rd version:

$$\underline{(4)}_h \text{ Find } \underline{u}_h = [u^{(i)}]_{i \in \bar{\omega}_h} \in \mathbb{R}^{\bar{N}_h}: K_h \underline{u}_h = \underline{f}_h \text{ in } \mathbb{R}^{\bar{N}_h}$$