

b) Assembling of the stiffness matrix $\hat{K}_h = [\hat{K}_{ij}]_{i,j \in \bar{\omega}_h}$

■ Starting point: $\forall i, j \in \bar{\omega}_h$

$$\hat{K}_{ij} = a(p^{(j)}, p^{(i)}) = \begin{cases} 0, & \text{if } B_{ij} = B_i \cap B_j = \emptyset \\ \sum_{r \in B_{ij}} \int_{\delta_r} (\lambda \nabla^T p^{(j)} \nabla p^{(i)} + \alpha p^{(j)} p^{(i)}) dx + \int_{\partial \delta_r} p^{(j)} p^{(i)} ds, & \text{otherwise} \end{cases}$$

model problem
from Subsection 2.2.1

$\Gamma_3 \cap \partial \delta_r$
3rd Kind BC
 $\rightarrow c)$

■ FE technology for elementwise computing the contributions to \hat{K}_{ij} :

$$\hat{K}_{ij} = \sum_{r \in B_{ij}} \int_{\delta_r} (\lambda \nabla^T p^{(j)} \nabla p^{(i)} + \alpha p^{(j)} p^{(i)}) dx \Rightarrow \hat{K}_h = [\hat{K}_{ij}]_{i,j \in \bar{\omega}_h}$$

$\forall i, j \in \bar{\omega}_h = \omega_h \cup \delta_h : B_{ij} := B_i \cap B_j \neq \emptyset$

NOT componentwise, BUT elementwise !!

Example: $i=8, j=9 \Rightarrow B_{ij} = \{8, 9, 18, 19, 20\} \cap \{9, 10, 20\} = \{9, 20\}$

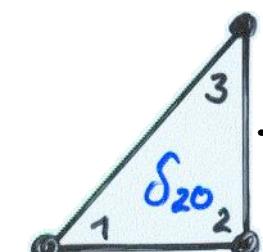
$$\hat{K}_{8,9} = \int_{\delta_9} (\dots) dx + \int_{\delta_{20}} (\dots) dx$$

Loop over all elements: $r = 1, \dots, 8, 9, 10, \dots, 19, 20, 21, \dots, 24$

(14)

$r: \alpha \leftrightarrow i$

1 2 3
8 9 14



element
Stiffness matrix

$$K^{(20)} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

$$\begin{array}{c} \oplus \\ \left[\begin{array}{ccc} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{array} \right] \\ \oplus \\ \left[\begin{array}{cc} R_{88} & R_{89} \\ R_{98} & R_{99} \end{array} \right] \end{array}$$

Assembling

$$\hat{K}_h \quad \tilde{N}_h \cdot K_h$$