- The equivalent to $(27)_{MP} \equiv (27)_{VP}$ <u>dual VP</u> can be formulated as follows:

$$(27)_{DVP} \quad \text{Find } w \in W: \; G(w) = \sup_{v \in W} G(v)$$

with the dual (complementary) energy functional (Trefftz)

$$(28) \quad G(v) = -\tfrac{1}{2} a(v,v) + a(g,v) - \langle F, g \rangle$$

and the linear manifold W of all solutions of $(27)_{VP}$ in $\overline{V}$, i.e.

$$W = \{ w = \bar{w} + v_0 : v_0 \in \overline{U}_0 \} = \{ w \in \overline{V} : a(w,v) = \langle F, v \rangle \; \forall v \in \overline{V}_0 \}$$

$$\overline{U}_0 = \{ v_0 \in \overline{V} : a(v_0, v) = 0 \; \forall v \in \overline{V}_0 \}$$

$$\bar{w} \in \overline{V} : a(\bar{w}, v) = \langle F, v \rangle \; \forall v \in \overline{V}_0$$

- Then we obtain by simple calculations (mms)

$$2 \| v - w \|^2 = E(v) - G(w) \; \forall v \in \overline{V}_g \; \forall w \in W$$

$$2 \| v - u_* \|^2 = E(v) - E(u_*) \; \forall v \in \overline{V}_g$$

$$2 \| w - u_* \|^2 = E(u_*) - G(w) \; \forall w \in W,$$

$$W \cap V_g = \{ u_* \},$$

where $\| \cdot \|^2 := a(\cdot, \cdot)$ denotes the energy norm. Therefore, the MAXIMUM and the MINIMUM of the TREFFTS- and the RITZ-functionals will be realized exactly for $u = w = u_*$, and the strong duality principle is valid, i.e.

$$(29) \quad \max_{v \in W} G(v) = G(u_*) = E(u_*) = \min_{v \in \overline{V}_g} E(v).$$