1.3.2. Dual Variational Formulations

- Let us first consider the minimum problem

$(27)_{MP}$ Find $u \in V_g$: $E(u) = \inf_{v \in V_g} E(v)$

with the primal energy functional (Ritz-functional)

$$(28) \quad E(v) := \frac{1}{2} a(v,v) - \langle F, v \rangle ,$$

and let us assume that

- $V_g = g + V_0 = \{ u \in V : u = g + v, \, v \in V_0 \}$
  $=$ linear manifold $=$ hyperplane
- $V_0 \subset V$ – closed subspace of the Hilbert-space $V, \|\cdot\|, (\cdot,\cdot)$,
- Standard assumption ($\rightarrow$ LAX-MILGRAM: 1), 2), 2a), 2b)),
- $a(\cdot,\cdot)$ is symmetric on V_0.

Then the MP $(27)_{MP}$ is equivalent to the VP

$(27)_{VP}$ Find $u \in V_g$: $a(u,v) = \langle F, v \rangle \;\; \forall v \in V_0$,

and, thanks to the LAX-MILGRAM-theorem, it has a unique solution $u \in V_g$.