Mixed VF of the 1st biharmonic BVP:

- <u>Idea:</u> CIARLET–RAVIART (1974), $\Omega \subset \mathbb{R}^2$, $\Gamma = \partial\Omega$

(26)
$$\Delta^2 u = f \text{ in } \Omega, \quad u = 0 \text{ on } \Gamma, \quad \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma$$

$$\to \quad w - \Delta u = 0 \text{ in } \Omega \quad \Big| \int_\Omega \cdot\, v\,dx \;\; \forall v \in V$$
$$-\Delta w = -f \text{ in } \Omega \quad \Big| \int_\Omega \cdot\, q\,dx \;\; \forall q \in Q$$
$$u = 0 \text{ on } \Gamma$$
$$\frac{\partial u}{\partial n} = 0 \text{ on } \Gamma \quad \text{nat. BC!}$$

$V = H^1(\Omega)$
$Q = \overset{\circ}{H}{}^1(\Omega)$

- $\displaystyle\int_\Omega w \cdot v\,dx + \int_\Omega \nabla^T u \nabla v\,dx - \int_\Gamma \frac{\partial u}{\partial n} v\,ds = 0 \quad \forall v \in V$

- $\displaystyle\int_\Omega \nabla^T w \cdot \nabla q\,dx - \int_\Gamma \frac{\partial w}{\partial n} \underbrace{q}_{=0}\,ds = -\int_\Omega f q\,dx \quad \forall q \in Q$

- <u>Mixed Variational Formulation:</u> $f \in L_2(\Omega)$ given

$(26)_{MVF}$

Find $w \in V = H^1(\Omega)$ and $u \in Q = \overset{\circ}{H}{}^1(\Omega)$:

$$\underbrace{\int_\Omega w \cdot v\,dx}_{a(w,v)} + \underbrace{\int_\Omega \nabla^T u \cdot \nabla v\,dx}_{b(v,u)} = \underbrace{0}_{=\langle F, v\rangle_V} \quad \forall v \in V,$$

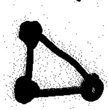
$$\underbrace{\int_\Omega \nabla^T w \cdot \nabla q\,dx}_{b(w,q)} = \underbrace{-\int_\Omega f \cdot q\,dx}_{=\langle G, q\rangle_Q} \quad \forall q \in Q,$$

- <u>Advantages / Disadvantages:</u>

⊕ Scalar 4th order PDE ⇒ System of 2 PDEs of 2nd order
→ i.e. only 1st order derivatives!

⊕ $\frac{\partial u}{\partial n}$ becomes natural BC!

⊕ Function $w = \Delta u$ ($\hat{=}$ bending moment in the plate bending, or vorticity in fluid mechanics) is directly computed!

⊕ FE-Galerkin-discretization with C^0 elements, e.g. 

⊖ Discretized problem is indefinit!