Exercises: for Section 1.2.3

$\boxed{\text{Ex 1.10}}$ = Exercise ~~12~~ 13

Show that the following properties hold for the first biharmonic BVP

(13) Find $\overset{\circ}{H}{}^2(\Omega)$: $\int_\Omega \Delta u \Delta v \, dx = \int_\Omega f v \, dx \quad \forall v \in \overset{\circ}{V}_0 = \overset{\circ}{H}{}^2(\Omega)$.

1.) $F \in V_0^* = H^{-2}(\Omega) = W_2^{-2}(\Omega)$,

2) $a(\cdot,\cdot): \overset{\circ}{V}_0 \times \overset{\circ}{V}_0 \to R^1$ is a bilinear form:

2a) $\exists \mu_1 = \text{const} > 0 : a(v,v) \geq \mu_1 \|v\|_2^2 \quad \forall v \in \overset{\circ}{V}_0 = \overset{\circ}{H}{}^2(\Omega)$,

2b) $\exists \mu_2 = \text{const} > 0 : |a(u,v)| \leq \mu_2 \|u\|_2 \|v\|_2 \quad \forall u,v \in \overset{\circ}{V}_0$,

2c) $a(u,v) = a(v,u) \quad \forall u,v \in \overset{\circ}{V}_0$.

These properties yield:

- $\exists!$ (Lax-Milgram) (Ass. 1) - 2b), but not 2c))
- Equivalence of $(13)_{VF}$ to the MP $(13)_{MP}$!

where $\|\cdot\|_2 := \|\cdot\|_{H^2(\Omega)}$.

$\boxed{\text{Ex 1.11}}$ = Exercise ~~13~~ 14

Derive the variational formulations of the BVP given in Remark 1.6.2! Investigate existence and uniqueness of generalized solutions (LGM)! Without loss of generality (homogenization), you can assume that the essential BC are homogeneous.