Fluid Mechanics: (→ Lectures "MathMod")

0) Stationary Navier-Stokes Equations:

describing the stationary flow of an incompressible Newtonian fluid ($d=3$):

$(22)_{CF}$ Find the velocity field $u(x) = (u_1(x), u_2(x), u_3(x))^T$ and the pressure field $p(x)$:

$$-\frac{1}{Re}\Delta u + \underbrace{(u\cdot\nabla)u}_{\text{non-linear convection term}} + \nabla p = f \text{ in } \Omega \subset \mathbb{R}^3,$$

$$\operatorname{div} u = 0 \text{ in } \Omega,$$

$+$ BC: e.g. $u = 0$ on $\Gamma = \partial\Omega$

VF ⇓ ①–⑤

$Re = \frac{\varrho}{\mu} \ell^* v^* = \frac{1}{\nu} \ell^* v^*$ – dimensionless Reynolds number

$(22)_{VF}$ Find $u \in V := (\overset{0}{H}{}^1(\Omega))^3$ and $p \in Q := \{q \in L_2(\Omega) : \int q \cdot 1 dx = 0\}$:

$$a(u;u,v) + b(v,p) = \langle F,v\rangle \quad \forall v \in V$$
$$b(u,q) = \langle G,q\rangle \quad \forall q \in Q$$

where $a(\cdot;\cdot,\cdot) : V \times V \times V \to \mathbb{R}^1$ – cont. trilinear form,

$$a(w;u,v) := \frac{1}{Re}\int_\Omega \nabla^T u \cdot \nabla v \, dx + \int_\Omega \sum_{i=1}^{3}\sum_{j=1}^{3} \underset{L_4}{w_j} \underset{L_2}{\frac{\partial u_i}{\partial x_j}} \underset{L_4}{v_i} \, dx, \quad \text{OK}$$

$b(u,q) := \int_\Omega \operatorname{div} u \cdot q \, dx$ – continuous bilinear form,

$\langle F,v\rangle := \int_\Omega f^T v \, dx, \quad G = 0$

Due to the convection term the N-S-problem (22) is non-linear. Solvability investigation ($\exists$ + !) is more difficult! The fix-point linearization leads to the so-called Ossen problem!