

Fluid Mechanics: (\rightarrow Lectures "MathMod")

o) Stationary Navier-Stokes Equations:

describing the stationary flow of an incompressible Newtonian fluid ($d=3$):

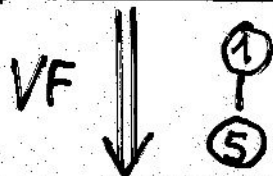
(22)
CF

Find the velocity field $u(x) = (u_1(x), u_2(x), u_3(x))^T$ and the pressure field $p(x)$:

$$-\frac{1}{\text{Re}} \Delta u + \underbrace{(u \cdot \nabla) u}_{\text{non-linear convection term}} + \nabla p = f \text{ in } \Omega \subset \mathbb{R}^3,$$

$$\text{div } u = 0 \text{ in } \Omega,$$

+ BC: e.g. $u = 0$ on $\Gamma = \partial\Omega$



$$\text{Re} = \frac{\rho}{\mu} l^{\nu} v^{\nu} = \frac{1}{\nu} l^{\nu} v^{\nu} - \text{dimensionless Reynolds number}$$

(22)
VF

Find $u \in V := (\dot{H}^1(\Omega))^3$ and $p \in Q := \{q \in L_2(\Omega) : \int_{\Omega} q \cdot 1 dx = 0\}$,

$$a(u; u, v) + b(v, p) = \langle F, v \rangle \quad \forall v \in V$$

$$b(u, q) = \langle G, q \rangle \quad \forall q \in Q$$

where $a(\cdot; \cdot, \cdot) : V \times V \times V \rightarrow \mathbb{R}^1$ - cont. trilinear form,

$$a(w; u, v) := \frac{1}{\text{Re}} \int_{\Omega} \nabla^T u \cdot \nabla v \, dx + \int_{\Omega} \sum_{i=1}^3 \sum_{j=1}^3 w_{ij} \frac{\partial u_i}{\partial x_j} v_i \, dx,$$

$L_4 \quad L_2 \quad L_4 \quad \text{OK}$

$b(u, q) := \int_{\Omega} \text{div } u \cdot q \, dx$ - continuous bilinear form,

$$\langle F, v \rangle := \int_{\Omega} f^T v \, dx, \quad G = 0$$

Due to the convection term the N-S-problem (22)

is non-linear. Solvability investigation (\exists +) is more difficult! The fix-point linearization leads to

the so-called Oseen problem!