1.3. Other Variational Formulations

1.3.1. Mixed Variational Formulations

- Mixed variational formulations of the form

(21)
Find $u \in V$ and $p \in Q$ such that

$$a(u,v) + b(v,p) = \langle F, v\rangle \quad \forall v \in V,$$
$$b(u,q) - c(p,q) = \langle G, q\rangle \quad \forall q \in Q,$$

where V, Q - Hilbert spaces, $F \in V^*$, $G \in Q^*$ given,

$\langle \cdot , \cdot \rangle_V : V^* \times V \longrightarrow \mathbb{R}^1$ - duality product,

$\langle \cdot , \cdot \rangle_Q : Q^* \times Q \longrightarrow \mathbb{R}^1$ - duality product,

$a(\cdot , \cdot) : V \times V \longrightarrow \mathbb{R}^1$ - non-negative, cont. bilinear f.

$b(\cdot , \cdot) : V \times Q \longrightarrow \mathbb{R}^1$ - continuous bilinear form,

$c(\cdot , \cdot) : Q \times Q \longrightarrow \mathbb{R}^1$ - non-negative, cont. bilinear f.

will be considered in the lectures "Computational
"CM" = Methods in Continuum Mechanics" (→ Zulehner),
and naturally arise in Fluid Mechanics

1) Oseen problem,
2) Stokes problem,

or artificially by introducing some dual variable

3) Hellinger-Reisner formulation of the Poisson equation or of the Linear 3D elasticity problem,
4) Mixed formulations of the plate bending problem.