Remark 1.5:

1. We refer to the Literature (e.g. [Ciarlet] pp. 23-28 or Lecture Notes on "Solid Mech.") for the discussion of existence and uniqueness of the solution of the mixed BVP (meas $\Gamma_1 > 0$ and meas $\Gamma_2 > 0$) and of the 2nd BVP ($\Gamma_2 = \Gamma$).
2. The basic tool for proving $\exists$ and $!$ is again the Lax-Milgram-Theorem.
   In order to prove V_0-ellipticity, we need
   - $D(\cdot)$ is uniformly SPD:
     $$\lambda_{min}(D)\, \|\varepsilon(v)\|^2_{L_2(\Omega)} \le a(v,v) \le \lambda_{max}(D)\, \|\varepsilon(v)\|^2_{L_2(\Omega)} \quad \forall v \in V$$
     with
     $$\lambda_{\substack{min\\max}}(D) = \substack{min\\max \\ x\in\overline{\Omega}}\ \underbrace{\lambda_{\substack{min\\max}}(x)}_{\text{EV of } D(x)},$$
   - <u>KORN's inequality</u>:
     $$(10) \qquad \|v\|_{H^1(\Omega)} \le c_K \Big[\sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|^2_{L_2(\Omega)} + \sum_{i=1}^{3} \|v_i\|^2_{L_2(\Omega)} \Big]^{1/2}$$
     $\forall v \in V := [H^1(\Omega)]^3$,
   - <u>FRIEDRICHS' inequality</u> (mixed BVP),
   - the knowlegde of the <u>rigid body motion subspace</u> (2nd BVP)
     $$\mathrm{Ker}\, a(\cdot,\cdot) = \mathrm{Ker}\, A_{Lamé}$$
     $$:= \{ a \times x + b : a, b \in \mathbb{R}^3 \}$$
     $$= \mathrm{span}\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}-x_2\\x_1\\0\end{pmatrix}, \begin{pmatrix}0\\-x_3\\x_2\end{pmatrix}, \begin{pmatrix}-x_3\\0\\x_1\end{pmatrix} \right\}$$
3. It holds (mms):
   $$(11) \qquad \varepsilon(v) = \mathbb{O} \iff v \in \mathrm{Ker}\, A_{Lamé}$$