→ T04d

■ RESULT: Variational Formulation (VF)

$(9)_{VF}$ = VF of (8)

Find $u \in V_g = V_0 := \{ v \in V = [H^1(\Omega)]^3 : v = 0 \text{ on } \Gamma_1 \}$:

$$a(u,v) = \langle F, v \rangle \quad \forall v \in V_0$$

where

$$a(u,v) = \int_\Omega \sum_{i,j=1}^{3} \sigma_{ij}(u(x))\, \varepsilon_{ij}(v(x))\, dx = \int_\Omega \sigma^T(u)\, \varepsilon(v)\, dx$$

$$= \int_\Omega \sum_{i,j=1}^{3} \sum_{k,l=1}^{3} D_{ijkl}\, \varepsilon_{kl}(u)\, \varepsilon_{ij}(v)\, dx = \int_\Omega \sigma^T(u)\, D\, \varepsilon(v)\, dx$$

tensor of elastic coefficients

$\boxed{\text{isotrop: } D_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})}$

$$= \int_\Omega \Big\{ \lambda \underbrace{\sum_{k=1}^{3} \varepsilon_{kk}(u)}_{= \operatorname{div} u} \underbrace{\sum_{i=1}^{3} \varepsilon_{ii}(v)}_{= \operatorname{div} v} + 2\mu \sum_{i,j=1}^{3} \varepsilon_{ij}(u)\, \varepsilon_{ij}(v) \Big\} dx$$

$$\langle F, v \rangle = \int_\Omega \sum_{i=1}^{3} f_i v_i\, dx + \int_{\Gamma_2} \sum_{i=1}^{3} t_i v_i\, ds = \int_\Omega f^T v\, dx + \int_{\Gamma_2} t^T v\, ds,$$

with given $f = (f_1, f_2, f_3)^T \in [L_2(\Omega)]^3$, $t = (t_1, t_2, t_3)^T \in [L_2(\Gamma_2)]^3$.

■ Minimization problem (MP):

$(9)_{MP}$

Find $u \in V_g$: $J(u) = \inf\limits_{v \in V_g} J(v)$

with the RITZ energy functional

$$J(v) = \underbrace{\frac{1}{2} \int_\Omega \sum_{i,j=1}^{3} \sigma_{ij}(v)\, \varepsilon_{ij}(v)\, dx}_{\substack{= \text{deformation energy} \\ \text{(inner energy)}}} - \underbrace{\Big(\int_\Omega f^T v\, dx + \int_{\Gamma_2} t^T v\, ds \Big)}_{\substack{= \text{potential energy} \\ \text{of the ext. forces}}}$$