Derivation of the VF in analogy to Section 1.2.1:

(1) $V_0 := \{ v = (v_1, v_2, v_3)^T \in V = [H^1(\Omega)]^3 : v = \mathbb{0} \text{ on } \Gamma_1 \}$

(2) $\int_\Omega [- \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} v_i] \, dx = \int_\Omega \sum_{i=1}^{3} f_i v_i \, dx \quad \forall v \in V_0$

(3) $\underbrace{\int_\Omega \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij} \frac{\partial v_i}{\partial x_j} dx}_{= \frac{1}{2}(\int_\Omega + \int_\Omega) = \int_\Omega \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij}(u) \varepsilon_{ij}(v) \, dx} - \int_\Gamma \sum_{i=1}^{3} \underbrace{\sum_{j=1}^{3} \sigma_{ij} n_j}_{=: \sigma_{n,i}} v_i \, ds = \int_\Omega f^T v \, dx$

$\sigma_{n,i}$ = i-th component of the normal stress vector

$\sigma_{ij} = \sigma_{ji}$

(4) $\int_\Gamma \sum_{i=1}^{3} \underbrace{[\sum_{j=1}^{3} \sigma_{ij} n_j]}_{\text{natural}} \underbrace{v_i}_{\text{essential}} \, ds =$

you can impose: natural essential boundary conditions

$= \int_{\Gamma_1} \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij} n_j \underbrace{v_i}_{=0} \, ds + \int_{\Gamma_2} \sum_{i=1}^{n} t_i v_i \, ds$

(5) $V_g := \{ v \in V : v = \bar{u} \text{ on } \Gamma_1 \} = V_0$

V_g = lin. manifold of admissible displacements

wlg: $\bar{u} = 0$ (otherwise: homogenization)

$(1)_g$ $\boxed{\text{Find } u \in V_g : \ a(u, v) = \langle F, v \rangle \ \forall v \in V_0}$

→ T04d