# Exercises: for Sect. 1.2.2

**Ex 1.7** Show that (8) is equivalent to LAMÉ's system of PDEs: (isotrop)

$$-\mu \Delta u(x) - (\lambda+\mu)\nabla \operatorname{div} u(x) = f(x),\ x\in\Omega,$$

+ BC: $u=\bar{u}$ on $\Gamma_1$ and $\sigma \cdot n = t$ on $\Gamma_2$,

with given $f=(f_1,f_2,f_3)^T$, $t=(t_1,t_2,t_3)^T$ and

$$\Delta = \begin{bmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{bmatrix} \text{ – vector Laplace,}$$

$\nabla$ = grad = gradient, div = divergence

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after T04d

**Ex 1.8** = Exercise 07 of Tutorial 2

Show that, for the first linear elasticity BVP, the following properties hold: $(\Gamma=\Gamma_1,\ \Omega\subset\mathbb{R}^3)$

1) $a(\cdot,\cdot)$ is symmetric, i.e. $a(u,v)=a(v,u)\ \forall u,v\in\bar{V}$,
2) $a(\cdot,\cdot)$ is non-negative, i.e. $a(v,v)\geq 0\ \forall v\in\bar{V}$,
3) $a(\cdot,\cdot)$ is positive on $V_0=\{v\in\bar{V}=[H^1(\Omega)]^3: v=0 \text{ on } \Gamma_1\}$,
   i.e. $a(v,v)>0\ \forall v\in V_0: v\neq 0$.

Properties 1) and 2) yield the equivalence of the VF $(9)_{VF}$ and the MP $(9)_{MP}$.

**Ex 1.9** = Exercise 08 of Tutorial 2

Show that, for the first linear elasticity BVP $(\Gamma_1=\Gamma)$, in the case of isotropic and homogeneous materials, the assumptions of Lax-Milgram's theorem are fulfilled! Compute $\mu_1$ and $\mu_2$!

**Hints** for proving the $V_0$-ellipticity:

✓ 1) $a(v,v)\geq 2\mu\int_\Omega \sum_{i,j=1} (\varepsilon_{ij}(v))^2 dx$

⇒ 2) KORN's inequality: $\int_\Omega \sum_{i,j} (\varepsilon_{ij}(v))^2 dx \geq c_K \|v\|^2_{H^1(\Omega)}\ \forall v\in V_0$ (?)

✓ 3) Friedrich's inequality.