$\boxed{\text{Ex } 1.5}$ = Exercise 07* of Tutorial 1

Derive the variational formulation of the Dirichlet problem for the Helmholtz equation

$$(**) \begin{cases} -\Delta u(x) - \omega^2 u(x) = f(x), \; x \in \Omega, \\ u(x) = 0, \; x \in \Gamma = \partial\Omega. \end{cases}$$

Then discuss the problem of the existence and the uniqueness of the generalized solution of the BVP $(**)$, where ω^2 is a given positive constant!

$\boxed{\text{Ex } 1.6}$ Let us consider the Dirichlet BVP for determining the z-component $u(x_1,x_2) = A_z(x,y)$ of the magnetic vector potential for a plane magnetic field problem (e.g. electrical machine):

$$\begin{cases} -\operatorname{div}\left(\frac{1}{\mu(x)} \nabla u(x)\right) = J_3(x) - \frac{\partial H_2(x)}{\partial x_1} + \frac{\partial H_1(x)}{\partial x_2}, \\ \qquad\qquad\qquad\qquad x = (x_1,x_2) \in \Omega \subset \mathbb{R}^2 \dagger, \\ + \text{BC: } u(x) = 0, \; x \in \Gamma = \partial\Omega \end{cases}$$

Derive the variational formulation:

$$\text{Find } u \in \tilde{V}_g : \; a(u,v) = \langle F, v \rangle \;\; \forall v \in \tilde{V}_0,$$

and show that there exists a unique generalized solution $u \in \tilde{V}_g = \tilde{V}_0$ provided that the following assumptions are fulfilled:

1) $\mu \in L_\infty(\Omega)$: $0 < \underline{\mu} \le \mu(x) \le \overline{\mu} \;\; \forall x \in \Omega$ with positive constants $\underline{\mu}$ and $\overline{\mu}$,
2) $J_3 \in L_2(\Omega)$,
3) $H_1, H_2 \in L_2(\Omega)$,
4) $\Omega \subset \mathbb{R}^2 \dagger$ and $\partial\Omega \in C^{0,1}$.