$\boxed{\text{Ex 1.3}}$ = Exercise 05 of Tutorial 1

In addition to the assumptions (?) let us assume that $c(x) \geq \underline{c} = const > 0 \;\; \forall x \in \Omega$, $\Gamma_1 = \Gamma_3 = \emptyset$, $b_i \not\equiv 0$.
Provide sufficient conditions for the coefficients b_i such that the assumtion of the L&M-Th. are satisfied!

<u>Hint:</u> For estimating the convection term
$\sum_i \int_\Omega b_i \frac{\partial u}{\partial x_i} v \, dx$, make use of the ε-inequality
$$|ab| \leq \frac{1}{2\varepsilon} a^2 + \frac{\varepsilon}{2} b^2 \quad !$$

$\boxed{\text{Ex 1.4}}$ = Exercise 06* of Tutorial 1

Derive the variational formulation of the pure Neumann problem for the Poisson equation
$$(*) \begin{cases} -\Delta u(x) = f(x), & x \in \Omega, \\ \frac{\partial u}{\partial n}(x) = 0, & x \in \Gamma = \partial\Omega, \end{cases}$$
and discuss the question of the existence and the uniqueness of the generalized solution of (*)!

<u>Hints:</u> Obviously, $u(x) + c$ with an arbitrary constant $c \in \mathbb{R}^1$ solves (*) provided that u is a solution of the BVP (*)!

There are the following ways to analyze (*):

1) Set up the variational formulation in $V = H^1(\Omega)$ and apply Fredholm's theory!
2) Set up the variational formulation in $V = H^1(\Omega)|_{Ker}$ with $Ker = \{c : c \in \mathbb{R}^1\} = \mathbb{R}^1$ and apply the Lax-Milgram-Theorem!
3) Consider the variational problem: Find $u \in V = H^1(\Omega)$:
$$\int_\Omega \nabla^T u \nabla v \, dx + \beta \int_\Omega u \, dx \int_\Omega v \, dx = \int_\Omega f v \, dx \quad \forall v \in V,$$
where $\beta \in \mathbb{R}^1_+$ is an arbitrary fixed positive constant.