Exercises:

Ex 1.1 = Exercises 01 - 02 of Tutorial 1

Formulate the classical assumptions which we have to impose on the data $\{a_{ij}, b_i, c, \alpha, f, g_i, \Omega\}$ of (5)! Provide sufficient conditions in order to ensure that a generalized solution $u \in V_g \cap X \cap H^2(\Omega)$ of (6) is also a solution of (5) in the classical sense!

Consider first the Dirichlet problem for the Poisson equation for the training:

$(5)_{\text{Poisson}}$

$$\boxed{\begin{array}{l} \text{Find } u \in X := C^2(\Omega) \cap C(\overline{\Omega}): \\ -\Delta u(x) = f(x), \; x \in \Omega \subset \mathbb{R}^d \; (*) \\ u(x) = g(x), \; x \in \Gamma = \partial\Omega \in C^2 \end{array}}$$

$$\overset{1}{\underset{5}{|}} \quad ? \quad \Downarrow \qquad\qquad \Uparrow \quad ? \quad \overset{1}{\underset{5}{|}}$$

$(6)_{\text{Poisson}}$

$$\boxed{\begin{array}{l} \text{Find } u \in V_g := \{v \in V = H^1(\Omega) : v = g \text{ on } \Gamma\}: \\ \underbrace{\int_\Omega \nabla^T u \, \nabla v \, dx}_{= a(u,v)} = \underbrace{\int_\Omega f v \, dx}_{= \langle F, v \rangle} \quad \forall v \in V_0 = \overset{\circ}{H}{}^1(\Omega) \end{array}}$$

Ex 1.2 = Exercise 03 : Show that in the following cases a) - c) the assumption of the L&M-Th. are satisfied, and compute μ_1 and μ_2! We assume (7) and

a) $b_i = 0$; $c(x) \geq 0 \;\; \dot{\forall} x \in \Omega$; $\alpha(x) \geq 0 \;\; \dot{\forall} x \in \Gamma_3$; $\text{meas}_{d-1}(\Gamma_1) > 0$;

b) $b_i = 0$; $c = 0$; $\alpha(x) \geq \underline{\alpha} = \text{const} > 0 \;\; \dot{\forall} x \in \Gamma_3$, $\text{meas}_{d-1}(\Gamma_3) > 0$, $\Gamma_1 = \emptyset$;

c) $b_i = 0$, $c(x) \geq \underline{c} = \text{const} > 0 \;\; \dot{\forall} x \in \Omega$, $\Gamma = \Gamma_2$ (i.e. $\Gamma_1 = \Gamma_3 = \emptyset$)