- **Remark 1.2:**

1. The solution $u \in \hat{V}_g$ of (6) is called weak or generalized solution.
2. The assumptions imposed on the data of (6) can be weakened (! integrals must exist !), e.g.

$$
(7)\left\{
\begin{array}{l}
1)\ a_{ij}, b_i, c \in L_\infty(\Omega),\ \alpha \in L_\infty(\Gamma_3), \\
2)\ f \in L_2(\Omega),\ g_i \in L_2(\Gamma_i),\ i = 2,3; \\
3)\ g_1 \in H^{1/2}(\Gamma_1) := \gamma_{0,\Gamma_1} H^1(\Omega), \text{ i.e. } \exists \tilde{g}_1 \in H^1(\Omega): \\
\quad \tilde{g}_1|_{\Gamma_1} := \gamma_{0,\Gamma_1}\tilde{g}_1 = g_1 \\
4)\ \Omega \subset \mathbb{R}^d\ \text{\textasteriskcentered}:\ \Gamma = \partial\Omega \in C^{0,1} \text{ (Lipschitz-cont.)} \\
5)\ \text{uniform ellipticity: } \exists \bar{\mu}_1 = \text{const} > 0: \\
\quad \left.\begin{array}{l} \sum_{i,j=1}^{d} a_{ij}(x)\xi_i\xi_j \geq \bar{\mu}_1 |\xi|^2 \ \forall \xi \in \mathbb{R}^d \\ a_{ij}(x) = a_{ji}(x)\ \forall i,j = \overline{1,d} \end{array}\right\} \dot{\forall} x \in \Omega \text{ a.e.}
\end{array}
\right.
$$

$*$:= bounded

$\dot{\forall}$ = for almost all

<u>Further weakening of the assumptions:</u> $d = 1,2,3$

$$
\left.\begin{array}{l} \text{e.g. } b_i \in H^1(\Omega) \Rightarrow b_i \in L_4(\Omega) \\ \quad v \in H^1(\Omega) \Rightarrow v \in L_4(\Omega) \end{array}\right\} \Rightarrow b_i v \in L_2(\Omega)
$$

3. <u>Relations between classical and generalized solutions:</u>

$$
\boxed{\begin{array}{c} u \in X : (5) \\ \text{classical solution} \end{array}}
\ \underset{\text{(1)} \longleftarrow \text{(5)}}{\overset{\substack{u \in X \cap V_g \cap H^2(\Omega) \\ \text{(1)} \longrightarrow \text{(5)}}}{\rightleftarrows}}\
\boxed{\begin{array}{c} u \in \hat{V}_g : (6) \\ \text{weak solution} \end{array}}
$$

<u>Ass.:</u>

- $u \in \hat{V}_g \cap X \cap H^2(\Omega)$
- classical assumptions imposed on the data in Ω

- Existence of the integrals and feasibility of the partial integration must be ensured!

! $u \in X = C^2(\Omega) \cap C^1(\Omega \cup \Gamma_2 \cup \Gamma_3) \cap C(\Omega \cup \Gamma_1) \not\Rightarrow u \in H^1(\Omega)$!
in general