

• Remark 1.2:

1. The solution  $u \in \tilde{V}_g$  of (6) is called weak or generalized solution.
2. The assumptions imposed on the data of (6) can be weakened (! integrals must exist!), e.g.

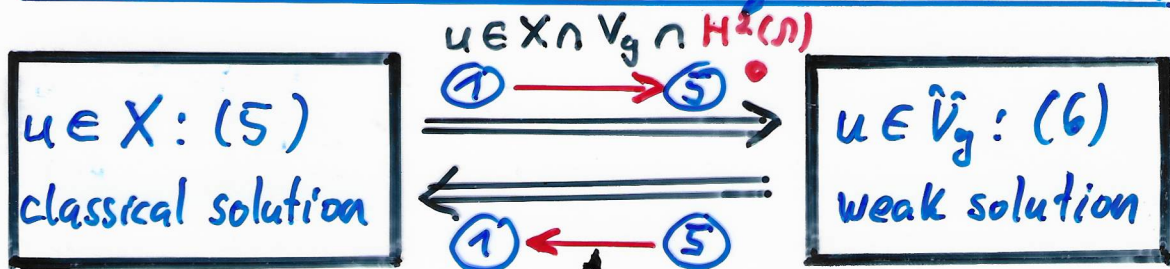
(7)

- 1)  $a_{ij}, b_i, c \in L_\infty(\Omega), \alpha \in L_\infty(\Gamma_3),$
- 2)  $f \in L_2(\Omega), g_i \in L_2(\Gamma_i), i=2,3;$
- 3)  $g_1 \in H^{1/2}(\Gamma_1) := \gamma_{0,\Gamma_1} H^1(\Omega),$  i.e.  $\exists \tilde{g}_1 \in H^1(\Omega):$   
 $\tilde{g}_1|_{\Gamma_1} := \gamma_{0,\Gamma_1} \tilde{g}_1 = g_1$
- 4)  $\Omega \subset \mathbb{R}^d \neq \emptyset: \Gamma = \partial\Omega \in C^{0,1}$  (Lipschitz-cont.)
- 5) uniform ellipticity:  $\exists \hat{\mu}_1 = \text{const} > 0:$   
 $\sum_{i,j=1}^d a_{ij}(x) \tau_i \tau_j \geq \hat{\mu}_1 |\tau|^2 \forall \tau \in \mathbb{R}^d$   
 $a_{ij}(x) = a_{ji}(x) \forall i,j = \overline{1,d}$

\* = bounded  
 \dot{\*} = for almost all

Further weakening of the assumptions:  $d=2,3$   
 e.g.  $b_i \in H^1(\Omega) \Rightarrow b_i \in L_4(\Omega)$   
 $v \in H^1(\Omega) \Rightarrow v \in L_4(\Omega)$  }  $\Rightarrow b_i v \in L_2(\Omega)$

3. Relations between classical and generalized solutions:



- $u \in \tilde{V}_g \cap X \cap H^2(\Omega)$
- classical assumptions imposed on the data in  $\Omega$

• Existence of the integrals and feasibility of the partial integration must be ensured!

•  $u \in X = C^1(\Omega) \cap C^1(\Omega \cup \Gamma_2 \cup \Gamma_3) \cap C(\Omega \cup \Gamma_1) \not\Rightarrow u \in H^1(\Omega)!$   
 in general