■ <u>BVP (5) is a mathematical model for</u>

1) Stationary heat conduction and heat transport

2) Stationary diffusion - convection problems

3) Potential problems:

   a) electrical scalar potential

     → Maxwell: $E = \nabla u$

$$\begin{cases} \text{Find } u \in X := C^2(\Omega) \cap C(\bar{\Omega}): \\ -\Delta u(x) = f(x), \ x \in \Omega, \\ \quad u(x) = g_1(x), \ x \in \Gamma_1 = \Gamma, \end{cases}$$

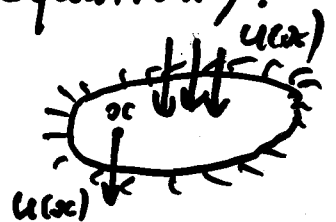   b) magnetic (vector) potential

     → Maxwell: $B = \text{curl } A$

   in 2D (e.g. cross-section of electrical machines):

$$\begin{cases} \text{Find } u = A_3 \in X := C^2(\Omega) \cap C(\bar{\Omega}): \\ -\text{div}\left(\frac{1}{\mu(x)} \nabla u(x)\right) = J_3(x) - \frac{\partial H_2}{\partial x_1} + \frac{\partial H_1}{\partial x_2}, \ x \in \Omega \subset \mathbb{R}^2, \\ + BC: \ u(x) = 0, \ x \in \Gamma_1 = \Gamma = \partial\Omega \end{cases}$$

4) Membran problem (= Poisson equation):

$$\begin{cases} \text{Find } u \in X: \ -\Delta u = f \ \text{in } \Omega \\ \qquad\qquad u = 0 \ \text{on } \Gamma \end{cases}$$

5) Helmholtz equation:

   → Periotic (harmonic) excitation
     of vibrations: $u(x,t) = u(x)e^{i\omega t}$

$$\begin{cases} \text{Find } u \in X := C^2(\Omega) \cap C(\bar{\Omega}): \\ -\Delta u(x) - K^2 u(x) = f(x), \ x \in \Omega, \\ \qquad\qquad u(x) = 0, \ x \in \Gamma_1 = \Gamma, \end{cases}$$

  ⋮   with $K^2 \approx \omega^2/a^2$.