## BVP (5) is a mathematical model for

1) Stationary heat conduction and heat transport
2) Stationary diffusion-convection-reaction problems
3) Potential problems:

a) electrical scalar potential

→ Maxwell: $E = \nabla u$

$$\begin{cases} \text{Find } u \in X := C^2(\Omega) \cap C(\overline{\Omega}): \\ -\Delta u(x) = f(x), \; x \in \Omega, \\ \qquad u(x) = g_1(x), \; x \in \Gamma_1 = \Gamma. \end{cases}$$

b) magnetic (vector) potential

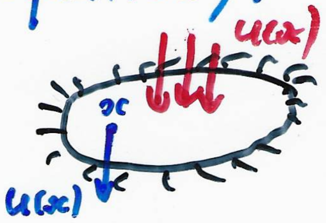
→ Maxwell: $B = \operatorname{curl} A$

in 2D (e.g. cross-section of electrical machines):

$$\begin{cases} \text{Find } u = A_3 \in X := C^2(\Omega) \cap C(\overline{\Omega}): \\ -\operatorname{div}\left(\frac{1}{\mu(x)} \nabla u(x)\right) = J_3(x) - \frac{\partial H_2}{\partial x_1} + \frac{\partial H_1}{\partial x_2}, \; x \in \Omega \subset \mathbb{R}^2, \\ + BC: \; u(x) = 0, \; x \in \Gamma_1 = \Gamma = \partial\Omega \end{cases}$$

4) Membran problem (= Poisson equation):

$$\begin{cases} \text{Find } u \in X: \; -\Delta u = f \text{ in } \Omega \\ \qquad u = 0 \text{ on } \Gamma \end{cases}$$




5) Helmholtz equation:

→ Periodic (harmonic) excitation of vibrations: $u(x,t) = u(x) e^{i\omega t}$

$$\begin{cases} \text{Find } u \in X := C^2(\Omega) \cap C(\overline{\Omega}): \\ -\Delta u(x) - \kappa^2 u(x) = f(x), \; x \in \Omega, \\ \qquad u(x) = 0, \; x \in \Gamma_1 = \Gamma, \end{cases}$$

with $\kappa^2 = \omega^2 / a^2$.

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