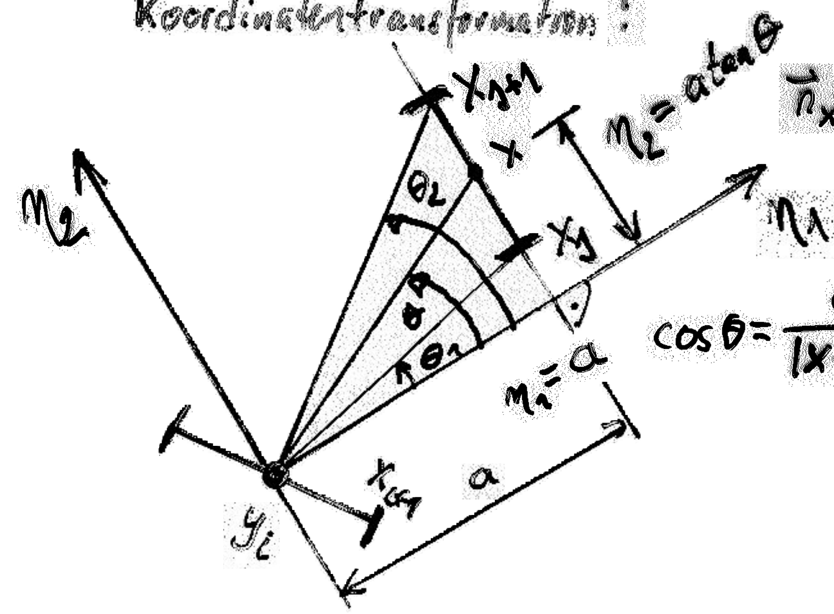
Berechnung von a_{ij} und $\tilde{b}_{ij}$ $(i,j = 1,n)$:

1. Sei $i \neq j$ $(i,j = 1,2,\dots,n)$:

$$a_{ij} = \int_{\Gamma_j} E(x,y_i)\,ds_x = -\frac{1}{2\pi}\int_{\Gamma_j} \ln|x-y_i|\,ds_x$$

Koordinatentransformation!

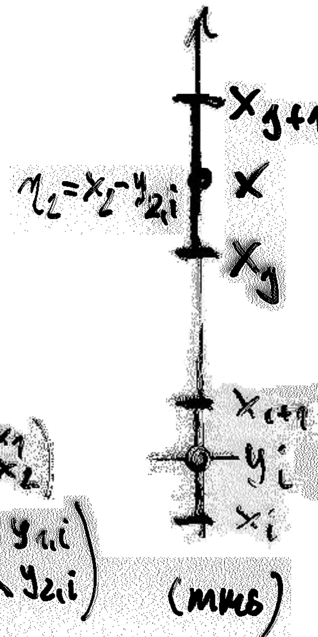


$\vec{n}_x = \begin{pmatrix}1\\0\end{pmatrix}$

$\cos\theta = \dfrac{a}{|x-y_i|}$

$x = \begin{pmatrix}x_1\\x_2\end{pmatrix}$

$y_i = \begin{pmatrix}y_{1,i}\\y_{2,i}\end{pmatrix}$



$$|x-y_i| = \frac{a}{\cos\theta}$$

$$ds_x = d\eta_2 = d(a\tan\theta) = \frac{a}{\cos^2\theta}\,d\theta = a\,\frac{d\tan\theta}{d\theta}\,d\theta$$

$$a_{ij} = -\frac{1}{2\pi}\int_{\theta_1}^{\theta_2} \ln\left(\frac{a}{\cos\theta}\right)\cdot\frac{a}{\cos^2\theta}\,d\theta = -\frac{a}{2\pi}\int_{\theta_1}^{\theta_2} \ln\frac{a}{\cos\theta}\cdot\frac{d}{d\theta}\tan\theta\,d\theta$$

$$= -\frac{a}{2\pi}\left[\ln\left(\frac{a}{\cos\theta}\right)\cdot\tan\theta\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\int_{\theta_1}^{\theta_2}\frac{\cos\theta}{a}\left(\frac{-a\cdot(-\sin\theta)}{\cos^2\theta}\right)\frac{\sin\theta}{\cos\theta}\,d\theta$$

$$= -\frac{a}{2\pi}\left[\ln\left(\frac{a}{\cos\theta}\right)\tan\theta\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\int_{\theta_1}^{\theta_2}\tan^2\theta\,d\theta$$

$$= -\frac{a}{2\pi}\left[\tan\theta\,\ln\frac{a}{\cos\theta}\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\left[\tan\theta - \theta\right]_{\theta_1}^{\theta_2} =$$

$$= -\frac{a}{2\pi}\left[\tan\theta\left(\ln\frac{a}{\cos\theta} - 1\right) + \theta\right]_{\theta_1}^{\theta_2} = \qquad g = \frac{a}{\cos\theta} = |x-y_i|$$

$$= -\frac{1}{2\pi}\Big[g\left(\sin\theta\,(\ln g - 1\right) + \theta\cos\theta\Big]_{g_1 = |x_j - y_i|,\ \theta_1}^{g_2 = |x_{j+1} - y_i|,\ \theta_2}$$