Differenzenschemata für Wärmeleitgl. (1):

$v(x,t) \approx u(x,t)$, $x = x_i = a + ih$, $i = \overline{0,n}$, $h = \Delta x = \frac{b-a}{n}$

$v_i^j = v(x_i, t_j)$ $\quad t = t_j = j\tau$, $j = \overline{0,m}$, $\tau = \Delta t = \frac{t_E}{m}$

- **Explizites Euler-Verfahren:** $(2) = (6)_{\theta=0}$

$$(2)\begin{cases} \underbrace{\dfrac{v_i^{j+1} - v_i^j}{\tau}}_{=:\, v_{t,i}^j} - \varkappa \underbrace{\dfrac{v_{i-1}^j - 2v_i^j + v_{i+1}^j}{h^2}}_{=:\, v_{\bar{x}x,i}^j} = f_i^j := f(x_i, t_j) \quad i = \overline{1,n-1},\ j = \overline{0,m-1} \\ \text{RB: } v_0^j = u_a(t_j),\ v_n^j = u_b(t_j),\ j = 0,1,\dots,m \\ \text{AB: } v_i^0 = u_0(x_i),\ i = 0,1,\dots,n-1,n \end{cases}$$

- **Implizites Euler-Verfahren:** $(5) = (6)_{\theta=1}$

$$(5)\begin{cases} \dfrac{v_i^{j+1} - v_i^j}{\tau} - \varkappa \dfrac{v_{i-1}^{j+1} - 2v_i^{j+1} + v_{i+1}^{j+1}}{h^2} = f_i^{j+1},\ j = \overline{0,m-1},\ i = \overline{1,n-1} \\ \text{RB: } v_0^j = u_a(t_j),\ v_n^j = u_b(t_j),\ j = 0,1,\dots,m \\ \text{AB: } v_i^0 = u_0(x_i),\ i = \overline{0,n} \end{cases}$$

- **θ-Verfahren:** $\theta \in [0,1]$

$$(6)\begin{cases} \dfrac{v_i^{j+1} - v_i^j}{\tau} - \theta \dfrac{v_{i-1}^{j+1} - 2v_i^{j+1} + v_{i+1}^{j+1}}{h^2} - (1-\theta) \dfrac{v_{i-1}^j - 2v_i^j + v_{i+1}^j}{h^2} = \\ \qquad = \theta f_i^{j+1} + (1-\theta) f_i^j,\ i = \overline{1,n-1},\ j = \overline{0,m-1} \\ + \text{RB } (\uparrow) + \text{AB } (\uparrow) \end{cases}$$

$\theta = \frac{1}{2}$: CRANK-NICOLSON (Trapezregel)