2. Mathematisch Korrekt!

$$z^{j+1} = \Big(I - \varkappa \frac{\tau}{h^2} \underbrace{\begin{bmatrix} -1 & 2 & -1 \end{bmatrix}}_{=A}\Big) z^j + \tau \psi^j \in \mathbb{R}^{n-1}$$

$$\|z\| \overset{z.B.}{:=} \|z\|_{\mathbb{R}^{n-1}} := \sqrt{\sum_{i=1}^{n-1} (z_i)^2}, \quad z \in \mathbb{R}^{n-1}$$

$$\|z^{j+1}\| \overset{\Delta\text{-Ungl}}{\le} \Big\|\Big(I - \varkappa\frac{\tau}{h^2}A\Big) z^j\Big\| + \tau\|\psi^j\|$$

$$\le \underbrace{\Big\|I - \varkappa\frac{\tau}{h^2}A\Big\|}_{=:q} \|z^j\| + \tau\|\psi^j\|$$

$$\|z^{j+1}\| \le q\|z^j\| + \tau\|\psi^j\| \le \dots \le$$

$$\le q^{j+1}\|z^0\| + \tau\big(q^j\|\psi^0\| + q^{j-1}\|\psi^1\| + \dots + \|\psi^j\|\big)$$

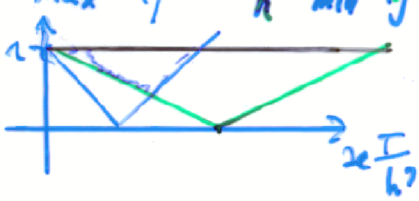
$$\le \|z^0\| + t_{j+1} \underbrace{\max_{k=0,j}\|\psi^k\|}_{O(\tau+h^2)} \le c(\tau+h^2)$$

Def

$$\boxed{\text{Stabilitätsbed.: } q = \Big\|I - \varkappa\frac{\tau}{h^2}A\Big\| \le 1}$$

$$q = \Big\|I - \varkappa\frac{\tau}{h^2}A\Big\| \overset{!}{=} \max\Big\{\Big|1-\varkappa\frac{\tau}{h^2}\lambda_{max}(A)\Big|, \Big|1-\varkappa\frac{\tau}{h^2}\lambda_{min}(A)\Big|\Big\}$$

Spektralnorm



$\lambda_{min/max}$ = min/max EW: $A\varphi = \lambda\varphi$

$$q < 1 \underset{(muss)}{\Longleftrightarrow} \Big|1 - \varkappa\frac{\tau}{h^2}\lambda_{max}(A)\Big| \le 1 \quad ✓$$

$$-1 \le 1 - \varkappa\frac{\tau}{h^2}\lambda_{max}(A) \le 1$$

$$-2 \le -\varkappa\frac{\tau}{h^2}\lambda_{max}(A)$$

$$\tau \le \frac{h^2}{\varkappa}\big(2/\lambda_{max}(A)\big)$$