$$\psi_i^j = \psi(x_i, t_j) = \underbrace{\left[\frac{u(x_i, t_j+\tau) - u(x_i, t_j)}{\tau} - \frac{\partial u}{\partial t}(x_i, t_j)\right]}_{= O(\tau)}$$

$$- \varkappa \underbrace{\left[\frac{u(x_i - h, t_j) - 2u(x_i, t_j) + u(x_i + h, t_j)}{h^2} - \frac{\partial^2 u}{\partial x^2}(x_i, t_j)\right]}_{= O(h^2)}$$

(3) $= O(\tau + h^2)$ = Approximationsfehler (Konsistenzfehler)

Resultat: (4)

$$\boxed{\begin{aligned} & z_{t,i}^j - \varkappa\, z_{\bar{x}x,i}^j = \psi_i^j := O(\tau + h^2) \\ & \text{RB: } z_0^j = z_n^j = 0 \quad \forall j = 0, 1, 2, \ldots, m \\ & \text{AB: } z_i^0 = u(x_i, t_0) - v_i^0 = 0, \quad i = \overline{0, n} \end{aligned}}$$

$$z_{t,i}^j = \varkappa\, z_{\bar{x}x,i}^j + \psi_i^j$$

(4) $$\boxed{\begin{aligned} & z_i^{j+1} = z_i^j + \varkappa \frac{\tau}{h^2}\left[z_{i-1}^j - 2z_i^j + z_{i+1}^j\right] + \tau \psi_i^j \\ & i = \overline{1, n-1}, \quad j = \overline{0, m-1} \end{aligned}}$$

(4) $$z^{j+1} := \begin{bmatrix} z_1^{j+1} \\ z_2^{j+1} \\ \vdots \\ z_{n-2}^{j+1} \\ z_{n-1}^{j+1} \end{bmatrix} = z^j - \varkappa \frac{\tau}{h^2} \begin{bmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 2 \end{bmatrix} z^j + \tau \psi^j$$