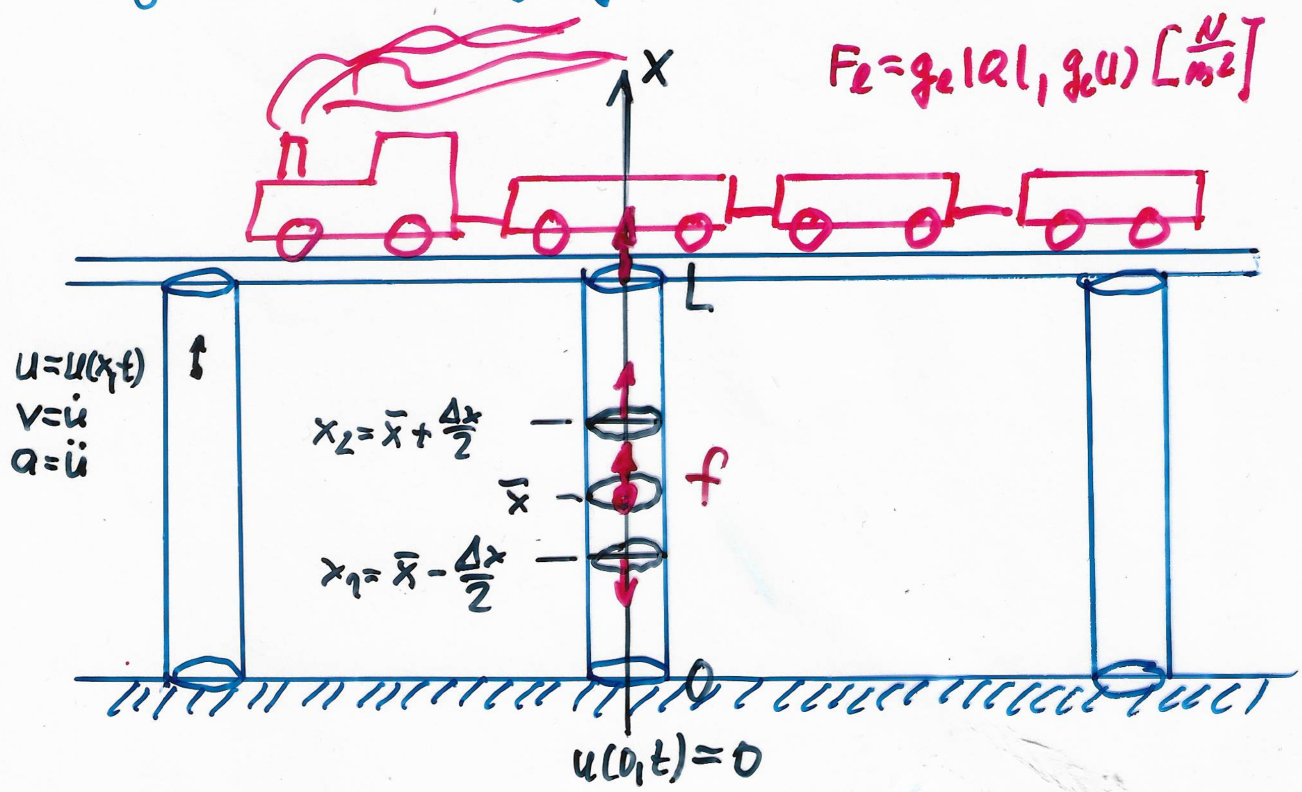
Mechanisches Problem: [JL 13: 2.2.2: 55-57]

Longitudinalschwingung eines Stabes (einer Säule)



Newton: Kraft = Masse × Beschleunigung

(*)
$$|Q|\,\sigma(\bar x+\tfrac{\Delta x}{2},t) - |Q|\,\sigma(\bar x-\tfrac{\Delta x}{2},t) + |Q|\int_{x_1}^{x_2} f(x,t)\,dx = \int_{x_1}^{x_2} a(x,t)\,\rho\,dx\,|Q|$$

resultierende Oberflächenkraft + resultierende Volumenkraft = resultierende Trägheitskraft

+ Stoffgesetz = HOOK : $\sigma = E\varepsilon$
+ Geometrische Beziehung: $\varepsilon(x,t) = u'(x,t) = \frac{\partial u}{\partial x}(x,t)$
+ $a(x,t) = \ddot u(x,t) := \frac{\partial^2 u}{\partial t^2}(x,t)$ – Beschleunigung
+ RB: $u(0,t) = 0,\ \sigma(L,t) := E\frac{\partial u}{\partial x}(L,t) = g_L(t)\ \forall t \in (0,t_E)$
+ AB: $u(x,0) = u_0(x),\ \frac{\partial u}{\partial t}(x,0) = v_0(x),\ \forall x \in [0,L]$

$\lim_{\Delta x \to 0} (*) \Longrightarrow$

$(\tilde{7})$

$$\rho\frac{\partial^2 u}{\partial t^2}(x,t) - \frac{\partial}{\partial x}\left(E\frac{\partial u}{\partial x}(x,t)\right) = f(x,t)\quad \forall (x,t) \in Q := (0,L)\times(0,t_E)$$

+ RB: $u(0,t) = 0,\ E\frac{\partial u}{\partial x}(L,t) = g_L(t)\quad \forall t \in (0,t_E)$
+ AB: $u(x,0) = u_0(x),\ \frac{\partial u}{\partial t}(x,0) = v_0(x),\ \forall x \in [0,L]$