ÜBUNGEN ZU

NUMERIK PARTIELLER DIFFERENTIALGLEICHUNGEN

für den 25. 1. 2006

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49. Show that the θ-method is A-stable if and only if $\theta \geq 1/2$.

50. Show: If a Runge-Kutta method is contractive for the differential equation

$$u'(t) = Ju(t),$$

it is also contractive for the differential equation

$$u'(t) = Ju(t) + f(t)$$

for an arbitrary function $f(t)$.

51. Let M and K are n-by$n$ matrices with $M^T = M > 0$ and $K^T = K > 0$. Show that the θ-method for $\theta < 1/2$ applied to the linear system

$$u'(t) = -M^{-1}Ku(t)$$

is contractive if

$$\tau \leq \frac{2}{(1-2\theta)\,\lambda_{\max}(M^{-1}K)}.$$

52. Let A be a real n-by-n matrix and let (z, z') be a scalar product in $\mathbb{C}^n$ with induced norm $\|z\| = (z,z)^{1/2}$. Show that

$$\sup_{0 \neq x \in \mathbb{R}^n} \frac{\|Ax\|}{\|x\|} = \sup_{0 \neq z \in \mathbb{R}^n} \frac{\|Az\|}{\|z\|}.$$

53. Show that a B-stable Runge-Kutta method is also A-stable.

Hint: Let $\lambda = \lambda_1 + i\lambda_2 \in \mathbb{C}$ with $\lambda_1 \leq 0$. Consider the linear system

$$\begin{pmatrix} u_1'(t) \\ u_2'(t) \end{pmatrix} = \begin{pmatrix} \lambda_1 & -\lambda_2 \\ \lambda_2 & \lambda_1 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix},$$

(which, of course, is equivalent to

$$u'(t) = \lambda\, u(t)$$

with $u(t) = u_1 + iu_2(t),$) and apply the definition of an B-stable method.

54. Show that the implicit Euler method is B-stable.

Hint: Show that

$$(w_{j+1} - v_{j+1}, w_{j+1} - v_{j+1}) \leq (w_j - v_j, w_{j+1} - v_{j+1}).$$