- **LAMÉsche Gleichungen im isotropen, homog. Fall:**

a) **Elastostatik:**

Substituieren: (33) → $(34)_{(ii)}$ → $(32)_{sta}$

$$\sigma_{ji} = \lambda\, \varepsilon_{kk}\, \delta_{ji} + 2\mu\, \varepsilon_{ji}$$

$$(36)\quad f_i(x) = -\sigma_{ji,j}(x) = -\lambda\, \varepsilon_{kk,i} - 2\mu\, \varepsilon_{ji,j}$$

$$= -\lambda \frac{\partial}{\partial x_i}\left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right] - \mu \frac{\partial}{\partial x_j}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right]$$

$$= -(\lambda+\mu)\frac{\partial}{\partial x_i}\left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right] - \mu \frac{\partial^2 u_i}{\partial x_j \partial x_i}, \quad \text{d.h.}$$

$$(36)_{sta}\quad \boxed{\begin{aligned}
-\mu\left[\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_1}{\partial x_3^2}\right] - (\lambda+\mu)\frac{\partial}{\partial x_1}\left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right] &= f_1 \\
-\mu\left[\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_3^2}\right] - (\lambda+\mu)\frac{\partial}{\partial x_2}\left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right] &= f_2 \\
-\mu\left[\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_3^2}\right] - (\lambda+\mu)\frac{\partial}{\partial x_3}\left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right] &= f_3
\end{aligned}}$$

$$(36)_{sta}\quad -\mu\, \Delta u(x) - (\lambda+\mu)\, \nabla\, \mathrm{div}\, u(x) = f(x), \quad x \in \Omega$$

$$(36)_{sta}\quad -2\mu\, \underbrace{\mathrm{div}\, \varepsilon(u)}_{\begin{bmatrix}\varepsilon_{1i}\\ \varepsilon_{2i}\\ \varepsilon_{3i}\end{bmatrix}} - \lambda\, \nabla\, \mathrm{div}\, u = f \text{ in } \Omega$$

Dazu kommen die RB (1.–4. Art) (↓)

b) **Elastodynamik:**

$$(36)_{dyn}\quad \varrho \frac{\partial^2 u_i}{\partial t^2}(x,t) - \mu \sum_{j=1}^{3} \frac{\partial^2 u_i}{\partial x_j^2}(x,t) - (\lambda+\mu)\frac{\partial}{\partial x_i}\sum_{j=1}^{3}\frac{\partial u_j}{\partial x_j}(x,t) = f(x,t)$$

$i = 1, 2, 3$ $\qquad (x,t) \in Q_T = \Omega \times (0,T)$

+ RB (1.–4. Art) (↓)

+ AB $u_i(x,0) = u_{0,i}(x,0)$, $\frac{\partial u_i}{\partial t}(x,0) = u_{1,i}(x,0)$, $x \in \overline{\Omega}$